Percentages

思考站

文具店正進行一年一度的大減價,所有文具均以八折出售,調皮的<u>小強</u>選購了 5 枝原子筆。

<u>小強</u>對店東説:「文具以八折出售,即每枝原子筆減價 20%,那麼 5 枝原子筆共減價:20% × 5 = 100%,對嗎?」

店東茫然地想:「不錯,20% × 5 = 100%,5 枝原子筆該減價 100%,我不應該收你的錢。如果每人都買 5 件文具,那我豈不是白做,怎麼辦?」

聰明的你快幫店東解決這個疑慮吧。

答案:
應以 5 件文具的總價來計算 20% 的折扣,即以不論買多少件,都只獲減價 20%。

應試錦囊

1. 比較兩數時,需注意何者是分母。
    例:有橙 10 個,蘋果 8 個,則

    (a) 橙比蘋果多 $= \dfrac{10-8}{8} \times 100\%$(蘋果的數目是分母)

    $\qquad\qquad\quad = 25\%$

    (b) 蘋果比橙少 $= \dfrac{10-8}{10} \times 100\%$(橙的數目是分母)

    $\qquad\qquad\quad = 20\%$

2. 已知某數的百分增加 (percentage increase) 或百分減少 (percentage decrease) 後的新值,求原值時需用除法或建立方程:
    例:某數增加 10% 後,新值為 220,求某數。

    方法 1:某數 $= 220 \div (1 + 10\%) = 200$

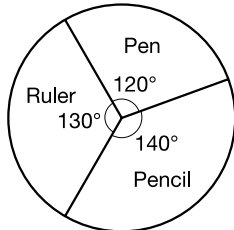   方法 2:設某數為 y。

    $\qquad\qquad y \times (1 + 10\%) = 220$

    $\qquad\qquad\qquad\qquad y = 200$

Warm Up Practice

1. Point out the mistakes in each of the following figures.

(a) Sales of stationeries

Pen
120°
Ruler
130°
140°
Pencil

(b) Ages of students

Number

11   12   13
Age

(c) Monthly savings
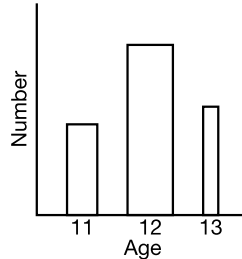
Savings ($)
500
100
50
20
10
0
J A S O N D
Month
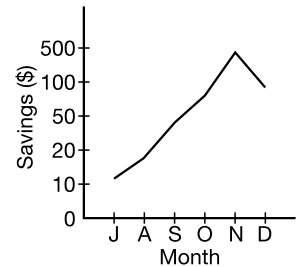
2. The following table shows the number of traffic accidents in a city.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Number of accidents	12	8	10	15	12	6	18

(a) Draw a bar chart to represent the data.

(b) On which day did the greatest number of accidents happen?

(c) What was the total number of accidents?

Solution:

(a)

Number of traffic accidents in a city

Number of accidents
20
15
10
5
0
Sun   Mon   Tue   Wed   Thu   Fri   Sat
Day
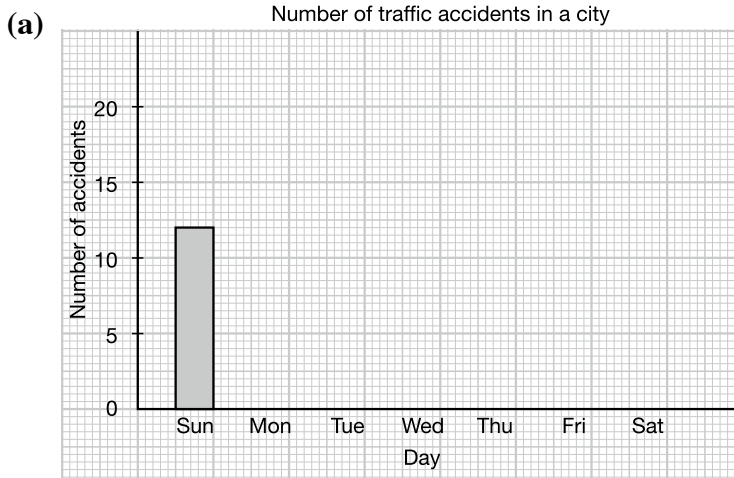
- 棒的高度應和數目成正比。
- 每枝棒的寬度應相等。

(b) The greatest number of accidents happened on

_____.

Test Your Understanding

Fundamental Stage

A. Multiple-choice Question

TSA 1. Which of the following is a polynomial?

A. $x^4 - x$ B. $\dfrac{x}{x^2 + 2}$

C. $\sqrt{x^4 + x^2}$ D. $4^x + x$

2. $2a + 4a + 5a =$
A. $9a$
B. $11a$
C. $11a^3$
D. $40a^3$

3. Simplify $a^6 \div a^6$.
A. 0
B. 1
C. a^2
D. a

4. $12a \div 2b \times 3c =$

A. $\dfrac{18a}{bc}$

B. $\dfrac{2ac}{b}$

C. $\dfrac{12ac}{b}$

D. $\dfrac{18ac}{b}$

5. $(3a)^2 \div 3a^2 =$
A. 1
B. 3
C. $3a^2$
D. $3a$

6. How many terms are there in the polynomial $2x + 3y - 4xy$?
A. 1 B. 2
C. 3 D. 4

7. What is the degree of the polynomial $2x^4 - 3x + 5$?
A. 1
B. 2
C. 3
D. 4

TSA 8. Which of the following polynomials has constant term equal to 0?
A. $(x + 2)(x + 3)$
B. $x(x + 3)$
C. $(x - 1)(x + 5)$
D. $x^2 + 0x + 6$

9. Consider the polynomial $2x^3 + 4x - 5$. Which of the following is not correct?
A. The degree is 3.
B. The constant term is 5.
C. The polynomial has 3 terms.
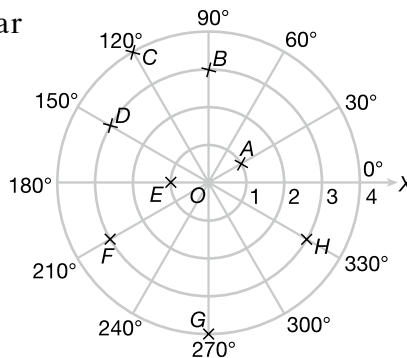D. The coefficient of x is 4.

10. Expand $(2x - 3y)^2$.
A. $4x^2 - 12xy + 9y^2$
B. $4x^2 - 12xy - 9y^2$
C. $4x^2 - 9y^2$
D. $4x^2 + 9y^2$

TSA 31. Referring to the figure, write down the polar coordinates of A to H.

32. The polar coordinates of P and Q are $(4, 30°)$ and $(6, 280°)$ respecitively. Find $\angle POQ$ (where O is the pole.)

33. The polar coordinates of A and B are $(6, 60°)$ and $(6, 120°)$ respectively. What kind of triangle is $\triangle AOB$?

34. The polar coordinates of T and S are $(2, 120°)$ and $(10, 120°)$ respectively. Find the length of TS.

TSA 35. Write down coordinates of the image A' of $A(2, -5)$ after each of the following transformations.
  (a) Translated to the right by 3 units
  (b) Reflected along the y-axis
  (c) Rotated about the origin through $180°$

Advanced Stage
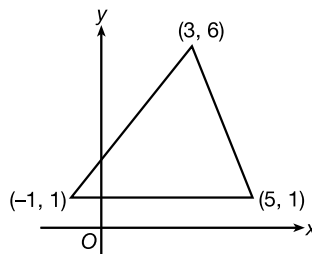
A. Multiple-choice Question

1. The vertices of a square are $(3, 0)$, $(0, 3)$, $(-3, 0)$ and $(0, -3)$. The area of the square is
  A. 9 sq. units.
  B. 16 sq. units.
  C. 24 sq. units.
  D. 36 sq. units.

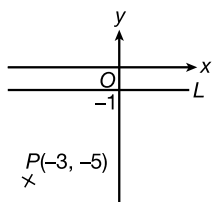**2.** The area of the triangle is

  A. 10 sq. units.
  B. 12 sq. units.
  C. 15 sq. units.
  D. 18 sq. units.

DSE 3. If the point $P(-3, -5)$ is reflected along the straight line L which is parallel to the x-axis, then the coordinates of the image of P are

A. $(-3, 1)$.
B. $(-3, 3)$.
C. $(-3, 5)$.
D. $(3, -5)$.

4. $A(1, 6)$, $B(1, 2)$ and $C(5, 2)$ are the vertices of a triangle. What kind of triangle will they form?

I.   Isosceles triangle
II. Right-angled triangle
III. Equilateral triangle

A. I only
B. II olny
C. III only
D. I and II only

5. Which of the following points is the farest from $P(-2, -4)$?

A. $Q(-2, 3)$
B. $R(-2, -8)$
C. $S(6, -4)$
D. $T(-9, -4)$

6. On the polar coordinate plane, which of the following point lies on the same circumference as $P(1, 30°)$?

A. $(1, 60°)$
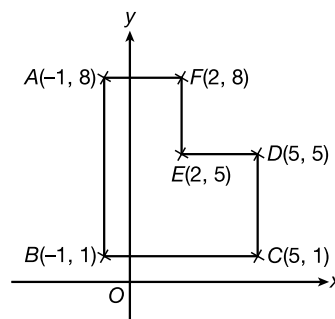B. $(2, 30°)$
C. $(2, 60°)$
D. $(3, 90°)$

B. Long Question

7. Find the area of each of the following figures formed by the given points.

(a) $A(3, 2)$, $B(7, 2)$, $C(7, 6)$ and $D(3, 6)$
(b) $P(-2, -3)$, $Q(4, -3)$, $R(6, 2)$ and $S(0, 2)$
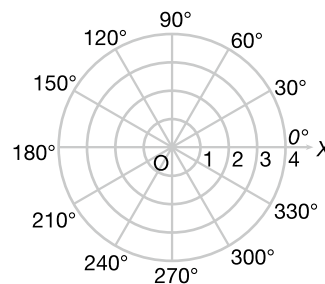
TSA 8. Find the area of the figure $ABCDEF$ in the rectangular coordinate plane.

Public Exam Classroom

Step-by-Step Demo

In a polar coordinate system, the polar coordinates of the points P, Q and R are (6, 72°), (8, 162°) and (9, 252°) respectively. Let O be the pole.

(a) Are P, O and R collinear? Explain your answer.

(b) Find the area of $\triangle PQR$.

(4 marks)

Solution:

(a) $\angle POR = 252° - 72° = 180°$

∴ P, O and R are collinear.

◀ 當三點所成的角為 180° 時，三點共線。 (1 M)
(1 M)

(b) Note that $QO \perp PR$.

$PR = (6 + 9)$ units = 15 units

Area of $\triangle PQR = \dfrac{15 \times 8}{2}$ sq. units

= 60 sq. units

◀ $\angle POQ = 162° - 72° = 90°$

◀ 利用三點共線及三點成一直角的關係，有助找出三角形的高及底，從而計算三角形的面積。 (1 M)
(1 M)

參考 Practice Paper Paper 1 Q6

Exam-type Question

1. In a polar coordinate system, the polar coordinates of the points A, B and C are (8, 125°), (9, 215°) and (10, 305°) respectively. Let O be the pole.

(a) Are A, O and C collinear? Explain your answer.

(b) What kind of triangle is $\triangle AOB$?

(c) Hence, find the area of $\triangle ABC$.

(5 marks)

2. The coordinates of the point P are $(-3, 4)$. P is rotated clockwise about the origin O through 90° to Q. R is the reflection image of P with respect to the x-axis.

(a) Write down the coordinates of Q and R.

(b) Find the area of $\triangle PQR$.

(5 marks)

◀ Open-ended Question

◀ 休憩室

Score sheet (Assessment 1)

Target I: To explore and manipulate directed numbers (Chapter 1)
Target II: To explore and understand the estimation technique (Chapter 2)
Target III: To understand the language of algebra and simple equation (Chapter 3)
Target IV: To recognize simple polynomial and the operation of polynomials (Chapter 4)
Target V: To understand percentage and its usage (Chapter 5)
Target VI: To recognize the simple method of data collecting in statistics (Chapter 6)
Target VII: To interpret and construct simple statistical graphs or charts (Chapter 7)

Questions / Target		I	II	III	IV	V	VI	VII	Total
Section A (30%)	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								
	11								
	12								
	13								
	14								
	15								
Section B (37%)	16								
	17								
	18								
	19								
	20								
	21								
	22								
Section C (33%)	23								
	24								
	25								
	Scores								
	☹	0 – 4	0 – 8	0 – 9	0 – 4	0 – 9	0 – 0	0 – 9	0 – 43
	😐	5 – 8	9 – 14	10 – 16	5 – 8	10 – 16	—	10 – 16	44 – 78
	😊	9 – 10	15 – 17	17 – 20	9 – 11	17 – 20	2 – 2	17 – 20	79 – 100

Key: ☹ Not Yet able to   😐 Beginning to develop the ability to   😊 Generally able to

Assessment 3 (Revision for Chapters 1 – 13)

Time allowed: 1 hour and 30 minutes
Full marks: 100
Answer ALL questions

Section A: Multiple-choice Question (30 marks)
Each question carries 2 marks.

1. Which of the following is false?
   A. $-4 + 5 = -1$
   B. $5 - (-4) = 9$
   C. $(-4) \times (+5) = -20$
   D. $\dfrac{-4}{5} = \dfrac{5}{-4}$

2. Express 'multiply the sum of 5 and y by 6' in an algebraic expression.
   A. $5(6 + y)$
   B. $5y + 6$
   C. $6(5 + y)$
   D. $5 + 6y$

3. What is the degree of the polynomial $3x^4 - 4x^2 + 5x - 1$?
   A. 1
   B. 2
   C. 3
   D. 4

4. Which of the following is not a linear equation in x?
   A. $3x - 15 = 0$
   B. $x^2 - 16 = 0$
   C. $\dfrac{x - 1}{2} = 5$
   D. $2(3 - x) = 24$

5. $33\dfrac{1}{3}\%$ of a year is equal to
   A. 3 months.
   B. 4 months.
   C. 6 months.
   D. 8 months.

6. Which strategy is the best to be used first in finding the approximate value of the expression
   $81.162 \times 19.56 \div 7.92$?
   A. Translation strategy
   B. Compensation strategy
   C. Round down
   D. Compatible numbers

TSA 7. A doctor conducted a survey and recorded the ages and average daily sleeping time of 10 patients. The doctor wants to use a statistical graph to find out whether the 2 sets of data relate to each other. Which of the following is the most suitable?
   A. Broken line graph
   B. Pie chart
   C. Scatter diagram
   D. Bar chart