Chapter 2

More about Percentages

思考站

某基金投資公司的李主任正在向陳先生推銷基金。

李主任：「我們現正代理兩種新推出的保證基金，回報率如下：

(1) 生物科技基金 —— 每年保證有 4% 的複式增長；

(2) 環球債券基金 —— 保證 3 年後有 15% 的增長。」

陳先生：「我打算投資三年，該買哪一種基金呢？」

答案：
$(1+4\%)^3 \approx 1.125 < 1 + 15\%$
∵ 故陳先生應購買 (2) 環球債券基金。

應試錦囊

1. 利息的利率一般以年利率 (interest rate per annum) 計算，所以時期需以年為單位。

2. 計算複利息 (compound interest) 時，若利息不是每年結算一次，則需注意在公式 $A = P \times (1 + r\%)^n$ 中，$r\%$ 及 n 的變化。

例：若年利率 $= 6\%$，年期 $= 3$ 年，複利息每半年結算一次，則

$r\% = 6\% \times \dfrac{6}{12} = 3\%$（因每次只可得到全年利息的一半）

$n = 2 \times 3 = 6$（因每年計算利息 2 次，共有 3 年）

3. 當情況複雜時，如有兩件或以上的事件時，我們可利用樹形圖 (tree diagram) 或列表法來協助找出所有可能結果 (possible outcome)。

4. 幾何概率 (geometric probability) 是關於幾何圖形的面積或長度等量度的概率。

Warm Up Practice

Important Formulas:

For an event E,

1. $P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

2. Experimental probability $= \dfrac{\text{Number of trials in which event } E \text{ occurs}}{\text{Total number of trials}}$

3. Geometric probability $= \dfrac{\text{Areas (/ length) of the region in which event } E \text{ occurs}}{\text{Areas (/ length) of the whole figure}}$

1. State whether each of the following statement is true (T) or false (F).

(a) If a card is drawn from a pack of poker at random, then the probability of getting a red card is the same as getting a black card.

(a) _____

(b) If a bag contains red balls only, then the event of getting a blue ball is an impossible event.

(b) _____

(c) $\dfrac{2}{\pi}$ can be the probability of an event.

(c) _____

(d) The experimental probability must be different from theoretical probability.

(d) _____

(e) When throwing a dice twice, the total number of possible outcomes is 36.

(e) _____

(f) If a bag contains \$5 coins only, then the event that a \$2 coin is taken out from this bag is an impossible event.
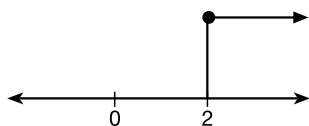
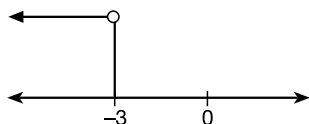(f) _____

Test Your Understanding

Fundamental Stage

A. Multiple-choice Question

1. The following graph represents the inequality

$$0 \qquad 2$$

A. $x > 2$.
B. $x \geq 2$.
C. $x < 2$.
D. $x \leq 2$.

2. The following graph represents the inequality

$$-3 \qquad 0$$

A. $x > -3$.
B. $x \geq -3$.
C. $x < -3$.
D. $x \leq -3$.

DSE 3. If $a > b > 0$, which of the following may not be true?

A. $-2a < -2b$
B. $\dfrac{a}{3} > \dfrac{b}{3}$
C. $(1-a)(1-b) < 0$
D. $1-a < 1-b$

4. How many positive integers satisfy the inequality $x \leq \dfrac{5\pi}{2}$?

A. 6
B. 7
C. 8
D. 9

5. Which of the following numbers does not satisfy the inequality $x > \sqrt{7}$?

A. 2.65
B. $\dfrac{8}{3}$
C. $\dfrac{3\pi}{2}$
D. $\dfrac{9}{4}$

DSE 6. Suppose $0 < a < b < 1$. Which of the following is false?

A. $ab > 1$
B. $2a < 2b$
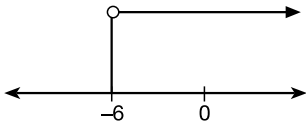C. $\dfrac{1}{a} > \dfrac{1}{b}$
D. $b - a > 0$

Advanced Stage

A. Multiple-choice Question

1. How many positive integers satisfy $\dfrac{x}{2} + \dfrac{x+1}{3} < \dfrac{x+2}{4}$?

A. 0
B. 1
C. 2
D. 3

2. a and b are solutions of the following figure, where $a < b$.

Which of the following must be true?

  I. $a + b > 0$
  II. $ab < 0$
  III. $(a + 6)(b - a) > 0$

A. I only
B. III only
C. I and II only
D. I and III only

DSE 3. If $a > b$ and a, b are both negative, which of the following is / are true?

  I. $2a < 2b$
  II. $-3a < -3b$
  III. $-3a > 3b$

A. II only
B. III only
C. I and III only
D. II and III only

4. If $ab > p$, which of the following must be true?

A. $b < p$
B. $a > \dfrac{p}{b}$
C. $ab + p > 0$
D. $ab + c > p + c$, where c is any number.

5. Solve $\dfrac{2x-1}{3} + \dfrac{1}{2} > \dfrac{1}{4}(x - 1)$.

A. $x < -1$
B. $x > -1$
C. $x < 1$
D. $x > 1$

6. If $a < b < 0$, which of the following is true?

A. $ab < 0$
B. $b - a < 0$
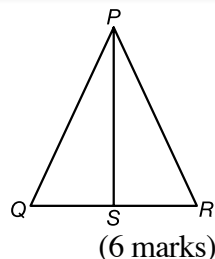C. $a + b < 0$
D. $a^2 < b^2$

Public Exam Classroom

考題趨勢

- DSE 卷二常見「三角形的中心」題型（常涉及其他課題的知識）
- DSE 沒有「三角形不等式」題型

Step-by-Step Demo

NF

In the figure, ΔPQR is an isosceles triangle with $PQ = PR$. S is a point on QR such that the orthocentre of ΔPQR lies on PS.

(a) Show that $\Delta PQS \cong \Delta PRS$.

(b) Are the orthocentre, circumcentre and incentre of ΔPQR collinear? Explain your answer.
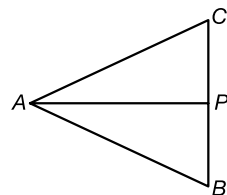
(6 marks)

Solution:

(a) $\because$ PS is an altitude.   ◄ 垂心為頂垂線的交點。

$\therefore$ $PS \perp QR$

$\therefore$ $\angle PSQ = \angle PSR = 90°$

$PQ = PR$ *(given)*

$PS = PS$ *(common side)* $\Big\}$ (2 M)

$\therefore$ $\Delta PQS \cong \Delta PRS$ *(RHS)*   (1 M)

(b) $QS = RS$ *(corr. sides, $\cong \Delta s$)*

Since PS is the perpendicular bisector of QR, the circumcentre lies on PS. $\Big\}$ (1 M)

$\angle QPS = \angle RPS$ *(corr. $\angle s, \cong \Delta s$)*

Since PS is the angle bisector of $\angle QPR$, the incentre lies on PS. $\Big\}$ (1 M)

$\therefore$ The orthocentre, circumcentre and incentre of ΔPQR are collinear.   (1 M)

參考 CE 2011 Paper 1 Q16

Exam-type Question

1. In the figure, $AB = AC$. P is a point on BC such that the
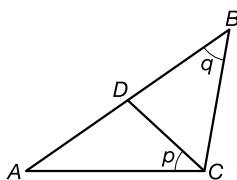NF incentre of ΔABC lies on AP.
   (a) Show that $\Delta ABP \cong \Delta ACP$.
   (b) Are the orthocentre, circumcentre and incentre of ΔABC collinear? Explain your answer.

(6 marks)

2. In the figure, CD is a median of ΔABC and $AD = CD$.
   (a) Find $p + q$.
   (b) Is ΔABC a right-angled triangle? Explain your answer.

(6 marks)

◄ Open-ended Question

◄ 休憩室

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Score sheet (Assessment 3)

Target I: To understand algebraic expression (Chapter 1, 3)
Target II: To apply percentages including growth and depreciation (Chapters 2)
Target III: To explore and study 3-D figures and their measurements (Chapters 4, 9)
Target IV: To study the probability and central tendency (Chapter 5, 6)
Target V: To recognize the deductive approach in handling geometric proofs (Chapters 7, 8)
Target VI: To apply trigonometric ratios, bearing and gradient (Chapters 10)
Target VII: To study the coordinate geometry of straight lines (Chapter 11)

Question	Target	I	II	III	IV	V	VI	VII	Total
Section A (30%)	1								
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								
	11								
	12								
	13								
	14								
	15								
Section B (37%)	16								
	17								
	18								
	19								
	20								
	21								
	22								
Section C (33%)	23								
	24								
	25								
	Scores								
	☹	0 – 5	0 – 4	0 – 7	0 – 6	0 – 9	0 – 6	0 – 9	0 – 46
	😐	6 – 9	5 – 7	8 – 12	7 – 11	10 – 16	7 – 10	10 – 16	47 – 81
	😊	10 – 11	8 – 9	13 – 15	12 – 14	17 – 19	11 – 13	17 – 19	82 – 100

Key: ☹ Not Yet able to   😐 Beginning to develop the ability to   😊 Generally able to

Assessment 3 (Revision for Chapters 1 – 11)

Time allowed: 1 hour and 30 minutes
Full marks: 100
Answer ALL questions

Section A: Multiple-choice Question (30 marks)
Each question carries 2 marks.

1. Which of the following is false?
   A. $a^2 - 25a = a(a - 5)$
   B. $a^2 - 49 = (a + 7)(a - 7)$
   C. $a^2 - 2a - 35 = (a + 5)(a - 7)$
   D. $a^3 + 10a^2 + 25a = a(a + 5)^2$

2. **(2.)** **NF** Which of the following has/have $x - 3$ as a factor?
   I. $x^3 - 27$
   II. $x^2 + 4x - 21$
   III. $x^4 - 81$
   A. I and II only
   B. I and III only
   C. II and III only
   D. I, II and III

3. Suppose $S = vt$. If v is increased by 10% and t is decreased by 8%, find the percentage change in S.
   A. −2.8%
   B. +1.2%
   C. +2%
   D. +2.5%

4. Tom is 10% heavier than Peter and Jack is 8% lighter than Peter. By how much percent is Tom heavier than Jack?
   A. 16.4%   (cor. to 1 d. p.)
   B. 17.2%   (cor. to 1 d. p.)
   C. 18%
   D. 19.6%   (cor. to 1 d. p.)

5. **(5.)** **NF** How many positive integers satisfy $x < \dfrac{6\sqrt{6}}{\sqrt{8}}$?
   A. 3
   B. 4
   C. 5
   D. 6

6. **TSA 6.** In the figure, a solid is formed by cutting out a cylinder from a cube with side length a. The base radius and the height of the cylinder are r and a respectively. Which of the following can be expressed by $6a^2 + 2\pi r(a - r)$?
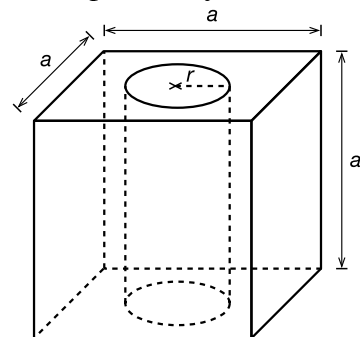
   A. Sum of the lengths of all edges of the solid
   B. Total surface area of the solid
   C. Base perimeter of the solid
   D. Volume of the solid