10. (a) Estimate the following sum by the method of compensation.

$$12.25 + 14.44 + 17.77 + 22.66$$

(b) Find the percentage error of the above estimation. (Give the answer correct to 2 significant figures.) (5 marks)

Suggested Solution

(a)
$$\begin{aligned} & 12.25 + 14.44 + 17.77 + 22.66 \\ \approx\ & 12 + 14 + 17 + 22 \\ =\ & 65 \end{aligned}$$ 1M

$$\begin{aligned} & 0.25 + 0.44 + 0.77 + 0.66 \\ =\ & 0.25 + 0.77 + 0.44 + 0.66 \\ \approx\ & 1 + 1 \\ =\ & 2 \end{aligned}$$ 1M

$$\begin{aligned} & 12.25 + 14.44 + 17.77 + 22.66 \\ \approx\ & 65 + 2 \\ =\ & \underline{\underline{67}} \end{aligned}$$ 1A

(b) The exact value of the sum $= 12.25 + 14.44 + 17.77 + 22.66$
$= 67.12$

The percentage error $= \dfrac{67.12 - 67}{67.12} \times 100\%$ 1M

$= \underline{\underline{0.18\%}}$ *(cor. to 2 sig. fig.)* 1A

Smart Tips

Step 1 is the rough estimation. We consider the integer parts only.

Smart Tips

Step 2 is the adjustment. We consider the decimal parts that are ignored in step 1.

Smart Tips

Step 3 is the adjusted estimation. We consider the results obtained in step 1 and step 2.

11. (a) Round up 154.45 to 1 significant figure.

Ref. *DSE 2014 Paper 1, Q.3*

(b) Round off 154.45 to the nearest integer.

(c) Round down 154.45 to 1 decimal place.

(3 marks)

Suggested Solution

(a) 200 1A

(b) 154 1A

(c) 154.4 1A

12. A pack of sugar is termed *regular* if its weight is measured as 50 g correct to the nearest g.

Ref. *DSE 2013 Paper 1, Q.8*

(a) Find the least possible weight of a *regular* pack of sugar.

(b) Is it possible that the total weight of 44 *regular* packs of sugar is measured as 2.1 kg correct to the nearest 0.1 kg? Explain your answer.

(5 marks)

Demonstration

- Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
- The diagrams are not necessarily drawn to scale.

Foundation Level 1

1. A factory produces 765 air-conditioners in 3 months. Find the production rate in each of the following units.

   (a) air-conditioners / month

   (b) air-conditioners / year

   (3 marks)

   Suggested Solution

   (a) The production rate

   $= \dfrac{765 \text{ air-conditioners}}{3 \text{ months}}$

   $= \underline{\underline{255 \text{ air-conditioners / month}}}$ 1A

   (b) The production rate

   $= \dfrac{765 \text{ air-conditioners}}{\frac{3}{12} \text{ year}}$ 1M

   $= \underline{\underline{3060 \text{ air-conditioners / year}}}$ 1A

2. ABC soft drink costs \$24 per pack. Each pack contains 8 cans and each can contains 355 mL. Find the price in each of the following units.

   (a) \$/can

   (b) \$/L

   (3 marks)

   Suggested Solution

   (a) The price

   $= \dfrac{\$24}{8 \text{ cans}}$

   $= \underline{\underline{\$3/\text{can}}}$ 1A

10. Suppose y^2 varies inversely as x^3, where x and y are positive numbers. Find the percentage change in y if x is increased by 25%. (4 marks)

Ref. **CE 2002 Paper 2, Q.15**

Strategies

We do not need to find the value of the variation constant.

Suggested Solution

Let $y^2 = \dfrac{k}{x^3}$, where k is a non-zero constant.

$$y = \sqrt{\frac{k}{x^3}} \qquad \text{1A}$$

Smart Tips

As y is a positive number, we can consider the positive square root only.

Let the new values of x and y be x' and y' respectively.

$$x' = (1 + 25\%)x$$
$$= 1.25x$$

$$y' = \sqrt{\frac{k}{(1.25x)^3}} = \sqrt{\frac{k}{1.25^3 x^3}} = \left(\sqrt{\frac{1}{1.25^3}}\right)(y) \qquad \text{1A}$$

$$\text{Percentage change} = \frac{y' - y}{y} \times 100\% \qquad \text{1M}$$

$$= \frac{\left(\sqrt{\frac{1}{1.25^3}}\right)(y) - y}{y} \times 100\%$$

$$= \left(\sqrt{\frac{1}{1.25^3}} - 1\right) \times 100\%$$

$= -28.4\%$ *(cor. to 3 sig. fig.)*

$\therefore$ y decreases by 28.4%. 1A

11. It is given that $f(x)$ is the sum of two parts, one part varies as x^3 and the other part is a constant. Suppose $f(-1) = 2$ and $f(-2) = -47$. Find $f(3)$. (4 marks)

Ref. **DSE 2014 Paper 1, Q.13(a)**

Suggested Solution

Let $f(x) = k_1x^3 + k_2$, where k_1 and k_2 are non-zero constants. 1A

$2 = k_1(-1)^3 + k_2$

$2 = -k_1 + k_2$ .... (1)

$-47 = k_1(-2)^3 + k_2$ 1M

$-47 = -8k_1 + k_2$ .... (2)

(1) – (2),

$49 = 7k_1$

$k_1 = 7$ 1A

Substituting $k_1 = 7$ into (1),

$2 = -7 + k_2$

$k_2 = 9$

$\therefore$ $f(x) = 7x^3 + 9$

$f(3) = 7(3)^3 + 9$

$= \underline{\underline{198}}$ 1A

Non-foundation Level 1

12. It is given that $f(x)$ partly varies directly as $(x+3)^2$ and partly varies directly as $(x+3)$. When $x=1$, $f(x)=56$. When $x=2$, $f(x)=90$.

Cross Functions and Graphs

(a) Express $f(x)$ as a polynomial in descending order of x. (4 marks)

(b) Find the minimum value of $f(x)$. (3 marks)

(c) Hence find the minimum value of $f(x-3)$. (1 mark)

Suggested Solution

(a) Let $f(x) = k_1(x+3)^2 + k_2(x+3)$, where k_1 and k_2 are non-zero constants.

Substituting $x=1$ and $f(x)=56$,

$$56 = k_1(1+3)^2 + k_2(1+3)$$
$$56 = 16k_1 + 4k_2$$
$$14 = 4k_1 + k_2 \text{.............. (1)}$$

Substituting $x=2$ and $f(x)=90$,

$$90 = k_1(2+3)^2 + k_2(2+3)$$
$$90 = 25k_1 + 5k_2$$
$$18 = 5k_1 + k_2 \text{.............. (2)}$$

1M

(2) – (1),

$k_1 = 4$ 1A

Substituting $k_1 = 4$ into (1),

$14 = 4(4) + k_2$

$k_2 = -2$ 1A

$\therefore\ f(x) = 4(x+3)^2 - 2(x+3)$
$= 4(x^2+6x+9) - 2x - 6$
$= 4x^2 + 24x + 36 - 2x - 6$
$= \underline{\underline{4x^2 + 22x + 30}}$ 1A

Watch Out

Students may forget to express the answer as a polynomial in descending order of x.

(b) Let (h, k) be the vertex of the graph of $y = f(x)$.

$h = \dfrac{-(22)}{2(4)} = -\dfrac{11}{4}$ 1M

$k = 4\left(-\dfrac{11}{4}\right)^2 + 22\left(-\dfrac{11}{4}\right) + 30$ 1M

$= -\dfrac{1}{4}$

$\therefore$ The minimum value of $f(x) = \underline{\underline{-\dfrac{1}{4}}}$ 1A

(c) The minimum value of $f(x-3)$

$=$ minimum value of $f(x)$

$= \underline{\underline{-\dfrac{1}{4}}}$ 1A

Senior Secondary

16B Basic Properties of Circles

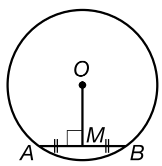
Key Point

Deductive Geometry
(Discussed in Chapter 16A)

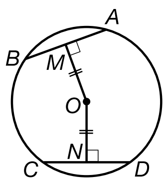
Circles

Chords of a Circle

1. (a) If $OM \perp AB$, then $AM = MB$.
   (b) Conversely, if $AM = MB$, then $OM \perp AB$.

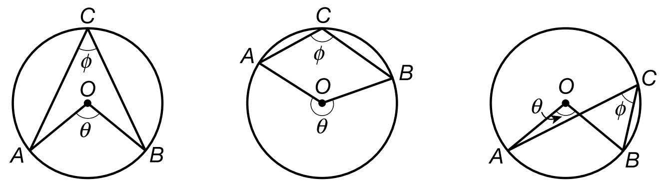


2. Given $OM \perp AB$ and $ON \perp CD$.
   (a) If $AB = CD$, then $OM = ON$.
   (b) Conversely, if $OM = ON$, then $AB = CD$.

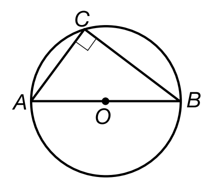


Angles of a Circle

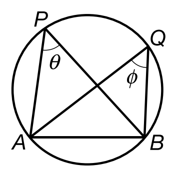
1. In each of the following figures, $\theta = 2\phi$.



2. (a) If AB is a diameter, then $\angle ACB = 90°$.
   (b) Conversely, if $\angle ACB = 90°$, then AB is a diameter.



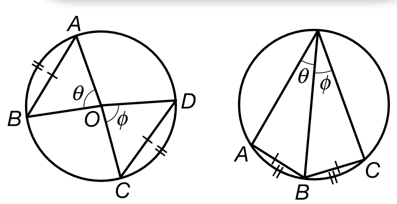
3. In the figure, $\theta = \phi$.



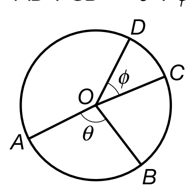
Relationship between the Chords, Arcs and Angles of a Circle

1.

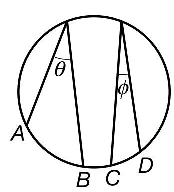
Equal arcs
Equal chords
Equal angles



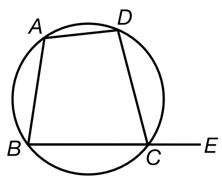
2. $\overparen{AB} : \overparen{CD} = \theta : \phi$



3. $\overparen{AB} : \overparen{CD} = \theta : \phi$



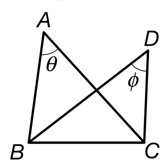
Cyclic Quadrilaterals



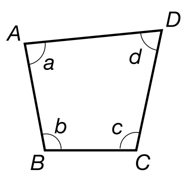
1. $\angle BAD + \angle BCD = 180°$ and $\angle ADC + \angle ABC = 180°$
2. $\angle DCE = \angle BAD$

Test for Concyclic Points

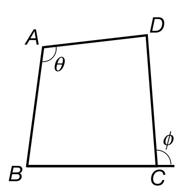
1. If $\theta = \phi$, then A, B, C and D are concyclic.



2. If $a + c = 180°$ (or $b + d = 180°$), then A, B, C and D are concyclic.

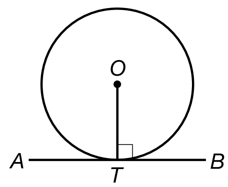


3. If $\theta = \phi$, then A, B, C and D are concyclic.

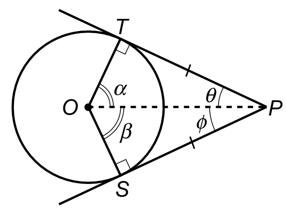


Tangents

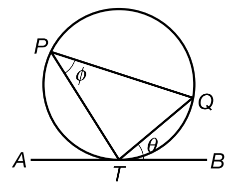
1. (a) If AB is a tangent to the circle at T, then $OT \perp AB$.
   (b) Conversely, if $OT \perp AB$, then AB is a tangent to the circle.



2. If PT and PS are tangents to the circle at T and S respectively, then
   (a) $PT = PS$,
   (b) $\theta = \phi$ and
   (c) $\alpha = \beta$.



3. Angle in Alternate Segment
   (a) If AB is a tangent to the circle at T, then $\theta = \phi$.
   (b) Conversely, if $\theta = \phi$, then AB is a tangent to the circle at T.

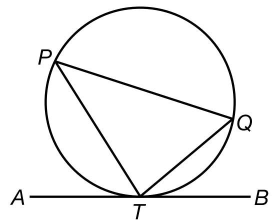


Senior Secondary

Non-foundation

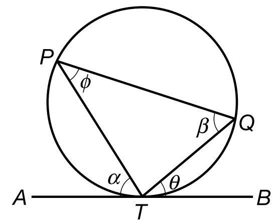
3. An angle included between a tangent and a chord of a circle is called a tangent-chord angle.

   In the figure, $\angle ATP$ and $\angle BTQ$ are tangent-chord angles.

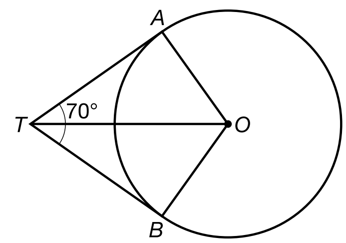


   (a) If AB is the tangent to the circle at T, then $\theta = \phi$ and $\alpha = \beta$.
   ($\angle$ in alt. segment)

   (b) Conversely, if $\theta = \phi$ (or $\alpha = \beta$), then AB is the tangent to the circle at T.
   (converse of $\angle$ in alt. segment)



   For example, in the figure, TA and TB are tangents to the circle at A and B respectively. If $\angle ATB = 70°$, find $\angle AOT$.



   Since TA and TB are tangents to the circle,

   $\angle ATO = \angle BTO$ *(tangent properties)*

   $\therefore \quad \angle ATO = \dfrac{70°}{2} = 35°$

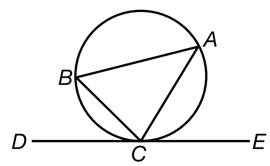
   $\therefore \quad \angle OAT = 90°$ *(tangent $\perp$ radius)*

   $\angle AOT + \angle OAT + \angle ATO = 180°$ *($\angle$ sum of Δ)*

   $$\begin{aligned}\angle AOT &= 180° - \angle OAT - \angle ATO \\ &= 180° - 90° - 35° \\ &= 55°\end{aligned}$$

Quick Check 10

In the figure, which angle equals $\angle BAC$?



Senior Secondary

DSE Exam Trend

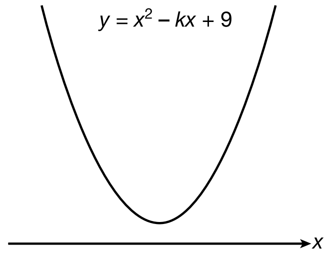
Recent HKDSE Questions:

	2014	2013	2012	Practice Paper	Sample Paper
HKDSE			8	7, 14	7, 19

Recent HKCEE Questions:

	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000
HKCEE				17	17	16	17	16	17	9, 16	5, 17	7, 16

34. The graph $y = x^2 - kx + 9$ lies above the x-axis. Find the range of possible values of k.

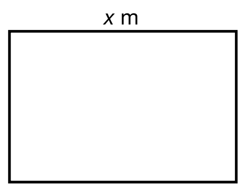


Ref. CE A Maths 2008, Q.4

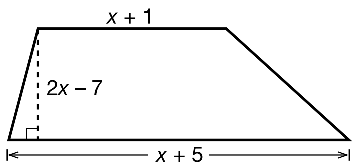
35. Let $f(x) = kx^2 - x + k$, where $k \neq 0$. Find the range of possible values of k such that $f(x) < 0$ for all real values of x. **Hint 4**

Ref. CE A Maths 2006, Q.4

36. David wants to fence in a rectangular field. It is given that the total length of the fence used is 100 m and the area of the field is at least 400 m^2. Let x m be the length of the rectangular field. Find the range of values of x and the maximum value of x.



37. The figure shows a trapezium with upper base, lower base and height equal to $(x + 1)$, $(x + 5)$ and $(2x - 7)$ respectively.

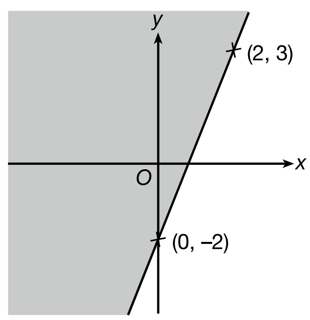


(a) Express the area of the trapezium in terms of x.

(b) If the area of the trapezium is less than 15 sq. units, find the range of values of x if x is an integer.

38. Solve $13x - y \leq 26$ graphically.

39. The figure shows the graph of a straight line which passes through (2, 3) and (0, –2).

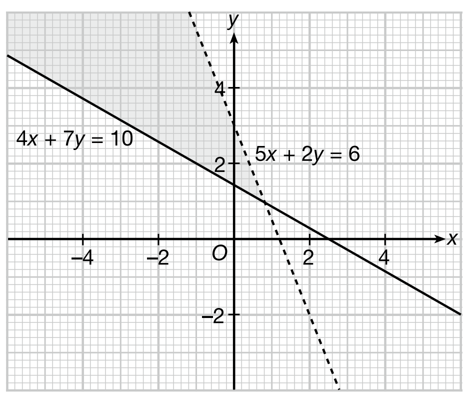


(a) Find the equation of the straight line. **Hint 5**

(b) Write down the inequality that is represented by the shaded region.

(c) Does (4, 4) satisfy the inequality in (b)?

40. Write down the system of inequalities that is represented by the shaded region.



Hint

Foundation Level 1

Hint 1 'A', 'E', 'I', 'O' and 'U' are vowels.

Foundation Level 2

Hint 2 The number of boys in the class changes when a new boy joins. Besides, the number of girls for selection is 20 as Mary will not be selected.

Non-foundation Level 1

Hint 3 Make use of the Venn Diagram to find the number of students who take only Biology.

Hint 4 Find the probability that the car park has a cover and the parking fee is \$8.

Hint 5 Note that 'only toy cars' and 'only toy ships' are not favourable outcomes.

Hint 6 If they have to sit next to each other, they can only sit at C1 and C2.

Hint 7 The probabilities of hitting each rectangular box are the same.

Hint 8 Express the given probabilities in terms of m and n.

Hint 9 Draw a tree diagram to list all of the possible outcomes.

Non-foundation Level 2

Hint 10 When he puts the ball into bag II, there are 10 balls in bag II. Consider the two cases that the ball from bag I is red and the ball is not red.

Hint 11 Consider all of the possible outcomes by using a table.

Hint 12 P(pass on the second attempt) = P(pass in two attempts) – P(pass on the first attempt)

Hint 13 P(different destinations) = 1 – P(same destination)

Hint 14 We only need to consider that Wilson travels by tram.

Hint 15 If his father is on duty, he is certainly not at home. The probability that his father is on duty is $\frac{2}{7}$.

Hint 16 Consider the two cases: (i) he studies; (ii) he does not study.

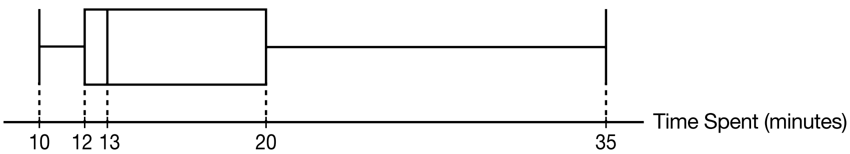
Hint 17 It is more convenient to analyze the cases by a tree diagram.

38. The stem-and-leaf diagram below shows the distribution of the time (in minutes) spent on the Internet daily by 20 secondary school students.

Stem (tens)	Leaf (units)
1	2 3 4 5 5
2	1 5 9
3	6 7 8
4	1 7 9 9 9
5	2 4 7 8

(a) Find the median, the range and the inter-quartile range of the distribution.

(b) The box-and-whisker diagram below shows the distribution of the time (in minutes) spent on the Internet by the 20 secondary school students during test period.

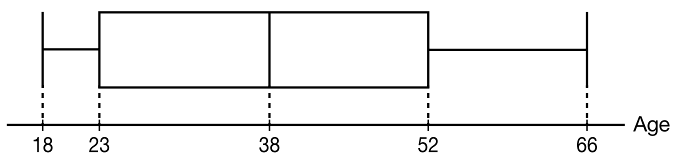


(i) Is the distribution of the time spent during the test period less dispersed than before? Explain your answer.

(ii) Someone claimed that none of the students had reduced their time spent on the Internet by 30 minutes or more during the test period. Do you agree? Explain your answer.

Ref. *CE 2009 Paper 1, Q.10*

39. The following box-and-whisker diagram shows the ages of the female members of a golf club.



The following are the statistics of the male members.

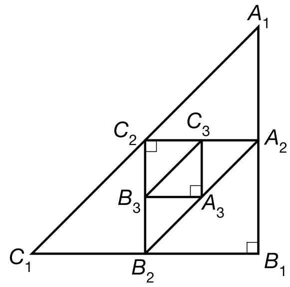
Minimum value	Lower quartile	Median	Upper quartile	Maximum value
a	25	38	b	68

(a) If the range and the inter-quartile range of the male members are 48 and 32 respectively, find a and b.

(b) Suppose that one man and one woman are selected randomly. Ken claims that a selected man will have a greatest chance of the age greater than 38. Do you agree? Explain your answer.

Ref. *ASL M & S 1999, Q.3*

70. $A_1B_1C_1$ is an isosceles triangle with $\angle B_1 = 90°$. The mid-points A_2, B_2 and C_2 of the sides of $\Delta A_1B_1C_1$ are drawn to form another triangle $A_2B_2C_2$. The process is continued to form triangles $\Delta A_3B_3C_3$, $\cdots$, $\Delta A_nB_nC_n$, $\cdots$.



(a) If $A_1B_1 = 8$ cm, find the area of $\Delta A_2B_2C_2$.

(b) Find the total area if n triangles are formed.

(c) Find the total area if the process continues infinitely.

71. The bank offers an interest rate of r% p.a. compounded half-yearly.

(a) A sum of money $\$P$ is deposited in a bank on the first day of each year. Find, in terms of P and r, the amount at the end of (Hint 39)

(i) the first year;

(ii) the second year;

(iii) the third year.

(b) Find the total amount after n years.

(c) Susan deposits \$30 000 in the bank at the beginning of each year with an interest rate of 9.5% p.a. compounded half-yearly. A car costs \$100 000 and its price increases by 12% each year. Show that Susan can afford to buy that car at the end of the fifth year. (Hint 40)

72. Figure 1 shows a right-angled triangle $A_1B_1C_1$ with $A_1B_1 = 10$ and $\angle C_1 = 30°$. A rectangle $A_2B_1B_2C_2$ is drawn inside $\Delta A_1B_1C_1$ with A_2 dividing A_1B_1 in the ratio 4 : 1.

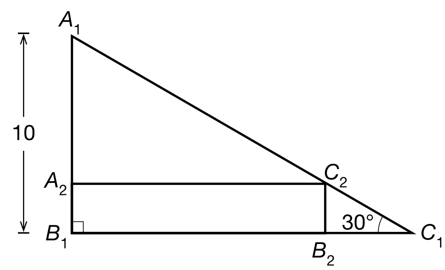


Figure 1

(a) (i) Find the length of B_1C_1.

(ii) Find the length of A_2C_2.

(Express the answers in surd form.)

(b) Another rectangle $A_3A_2B_3C_3$ is drawn inside $\Delta A_1A_2C_2$ with A_3 dividing A_1A_2 in the ratio 4 : 1. Suppose the process is repeated continually to the nth rectangle. Find the sum of the lengths of the n rectangles (i.e., find $A_2C_2 + A_3C_3 + \cdots$.).

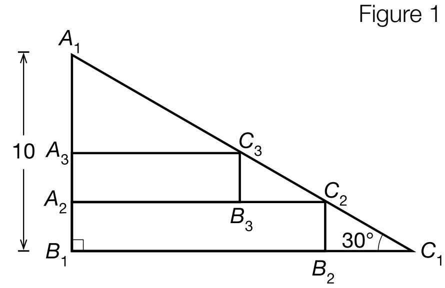


Figure 2

(c) Do the areas of the rectangles form a geometric sequence? Explain your answer.

73. Supermarket A sells goods of value $\$X(w)$ in the wth year since it opens, where w is a positive integer. It is given that the value of the goods sold by A in the 1st year and the 2nd year since it opened are \$1 505 280 and \$1 806 336 respectively. Suppose that $X(w) = \frac{1}{4}e^2 f^w$, where $e > 0$.

(a) (i) Find e and f. By using the values of e and f, find the value of the goods sold by A in the 5th year since it opened.

(ii) Express the total value of the goods sold by A in the first w years in terms of w.

(b) Another supermarket B starts to operate since A opened for 5 years. The value of the goods sold by B in the vth year since it opens, where v is a positive integer, is given by $\$Y(v)$. Suppose that $Y(v) = e^2 f^{2v}$.

(i) The consulting company claims that the value of goods sold by A is less than that by B in each year. Do you agree? Explain your answer.

(ii) New supermarket will be opened if the total value of the goods sold by A and B exceeds \$32 000 000 since their openings. In which year since supermarket A opened should the new supermarket be opened?

Ref. **DSE 2012 Paper 1, Q.19**

74. The total number of fans retained in a factory at the end of the first year is 88 000. The number of fans produced in each successive year will be r% of the total number of fans retained of the previous year. It is given that the number of fans which can be sold each year is 12 000. Suppose the total number of fans retained at the end of the third year is 121 120.

(a) (i) Express the total number of fans retained in the factory at the end of the 2nd year in terms of r.

(ii) Find r.

(b) (i) Express the total number of fans retained in the factory at the end of the nth year in terms of n.

(ii) At the end of which year will the total number of fans retained in the factory exceed 550 000?

(c) The demand of a country for the fans at the end of the nth year is $[x(1.69)^n + y]$, where x and y are constants. It is given that the demand of the country for the fans at the end of the 1st year and the 2nd year are 103 800 and 127 122 respectively. The total number of fans retained in the factory should be more than the demand of the country's needs. An officier of the country claims that the factory cannot fulfill the needs in every year. Do you agree? Explain your answer.

Ref. **DSE 2013 Paper 1, Q.19**