

# Chapter 3

## Percentages (2)



### 思考站

某基金投資公司的李主任正在向陳先生推銷基金。

李主任：「我們現正代理兩種新推出的保證基金，回報率如下：

- (1) 生物科技基金 —— 每年保證有 4% 的複式增長；
- (2) 環球債券基金 —— 保證 3 年後有 15% 的增長。」

陳先生：「我打算投資三年，該買哪一種基金呢？」



答案：  
 $(1+4\%)^3 \approx 1.125 > 1+15\%$   
∴ 應購買 (2) 環球債券基金



### 應試錦囊

1. 利息的利率一般以年利率 (interest rate per annum) 計算，所以時期需以年為單位。
2. 計算複利息 (compound interest) 時，若利息不是每年結算一次，則需注意在公式  $A = P \times (1 + r\%)^n$  中， $r\%$  及  $n$  的變化。  
例：若年利率 = 6%，年期 = 3 年，複利息每半年結算一次，則  
$$r\% = 6\% \times \frac{6}{12} = 3\% \text{ (因每次只可得到全年利息的一半)}$$
$$n = 2 \times 3 = 6 \text{ (因每年計算利息 2 次，共有 3 年)}$$
3. 計算定期存款時，需注意到期時是否提取利息。
  - (a) 若每次都提取利息，可視作單利息 (simple interest) 處理。
  - (b) 若將利息滾存下去，則可視作複利息處理。

4. 若  $P$  的值 3 年內每年增長 10% ，則 3 年後  $P$  的值不是  $P \times (1 + 10\% \times 3)$  ，應為  $P \times (1 + 10\%)^3$  。
5. (a) 在增長 (growth) 公式  $P' = P \times (1 + r\%)^n$  中 ，  
 $(1 + r\%)$  稱為增長因子 (growth factor) 。
- (b) 在折舊 (depreciation) 公式  $P' = P \times (1 - r\%)^n$  中 ，  
 $(1 - r\%)$  稱為衰變因子 (decay factor) 。
6. 求本金或原值時 ，可用除法或建立方程計算 。
7. 計算連續百分增加或減少時 ，因為原數不斷改變 ，所以不可以將所給的百分數相加或相減 。
- 例：照相機價錢 3 個月內分別下降了 5% 、 6% 及 8% ，總減少率不是  $5\% + 6\% + 8\% = 19\%$   
 設照相機的原價為  $y$  ，  
 則新價 =  $y(1 - 5\%)(1 - 6\%)(1 - 8\%) = 0.82156y$   
 減少率 =  $\frac{y - 0.82156y}{y} \times 100\% = 17.844\%$
8. 我們應依循以下步驟來計算薪俸稅 (salaries tax) ：
- (i) 計算免稅額 (allowance) ；
- (ii) 扣除免稅額 ，得出「應課稅入息實額 (net chargeable income)」；
- (iii) 將應課稅入息實額乘以稅率 ，便可求得需繳交的薪俸稅稅款 。



## Warm Up Practice

### Important Formulas:

1. Let  $P$  = principal,  $R\%$  = interest rate per period,  $T$  = number of periods,  $I$  = interest and  $A$  = amount.

(a) Simple Interest

$$I = P \times R\% \times T$$

$$A = P + I$$

**(b) Compound Interest**

$$A = P \times (1 + R\%)^T$$

$$I = A - P$$

2. Let  $P$  = original value and  $P'$  = new value.

(a) If  $P$  grows at a rate of  $r\%$ , then after  $n$  periods,  $P' = P \times (1 + r\%)^n$ .

(b) If  $P$  depreciates at a rate of  $r\%$ , then after  $n$  periods,  $P' = P \times (1 - r\%)^n$ .

1. State whether each of the following statements is true (T) or false (F).

(a) If  $A$  is greater than  $B$  by 10% and  $B$  is greater than  $C$  by 10%, then  $A$  is greater than  $C$  by 20%. (a) \_\_\_\_\_

(b) If the value of a machine depreciates 10% each year for 3 successive years, then its value will depreciate 30% in 3 years. (b) \_\_\_\_\_

(c) If the population grows 5% each year, then the growth factor is 1.05. (c) \_\_\_\_\_

(d) If  $P$  is increased to  $Q$ , then the percentage increase is  $\frac{Q - P}{Q} \times 100\%$ . (d) \_\_\_\_\_

(e) If the annual income of a person is less than the allowance, then he does not need to pay any salaries tax. (e) \_\_\_\_\_

2. If \$32 000 is deposited in a bank at an interest rate of 4% p.a. for 2 years, find the difference between the simple interest and the interest compounded yearly.

**Solution:**

Simple interest =  $\$32\,000 \times 4\% \times 2 =$  \_\_\_\_\_

Compound interest =  $\$32\,000 \times (1 + 4\%)^2 - \$$  \_\_\_\_\_  
= \_\_\_\_\_

Difference =  $\$($  \_\_\_\_\_  $-$  \_\_\_\_\_  $)$   
= \_\_\_\_\_

**Solution:**

Tax on the first \$50 000 =  $\$50\,000 \times 2\% =$  \_\_\_\_\_

Remaining part =  $\$(72\,000 - 50\,000) =$  \_\_\_\_\_

Tax on the remaining part = \_\_\_\_\_

= \_\_\_\_\_

Salaries tax payable = \_\_\_\_\_ + \_\_\_\_\_

= \_\_\_\_\_

- 先計算不同稅階的稅款，然後將它們相加。



## Test Your Understanding



### Fundamental Stage

#### A. Multiple-choice Question

- Mr. Chan deposits \$30 000 for 2 years at 6% p.a. compounded half-yearly. Which of the following is the interest that he will get?
  - $\$30\,000 \times 6\% \times 2$
  - $\$30\,000 \times (1 + 6\%)^2 - \$30\,000$
  - $\$30\,000 \times (1 + 6\%)^4 - \$30\,000$
  - $\$30\,000 \times (1 + 3\%)^4 - \$30\,000$
- If the principal \$ $P$  becomes \$ $Q$  in  $n$  years with simple interest rate  $r\%$  p.a., then  $r =$ 
  - $\frac{100(Q - P)}{nP}$
  - $\frac{100(Q - P)n}{P}$
  - $\frac{Q - P}{nP}$
  - $\frac{(Q - P)n}{P}$
- How long does it take for a principal to double itself at a simple interest rate of 20% p.a.?
  - 4 years
  - 5 years
  - 10 years
  - 20 years
- DSE** 4. \$6000 is put in a 3-month fixed deposit account at 8% p.a. Find the compound interest after 2 years, correct to the nearest dollar.
  - \$960
  - \$998
  - \$1030
  - \$7030

5. Which of the following is NOT necessary in calculating the compound interest?
- A. Growth rate  
 B. Interest rate  
 C. Principal   
 D. Time of deposit
6. The current price of a book is \$60. If the price increases by 10% each year, find the price of the book after 2 years.
- A. \$68.6                      B. \$72   
 C. \$72.6                      D. \$79.9
7. The value of a watch was increased by 10% each year during the past 3 years. If its present value is \$2662, what is the actual increase of its value?
- A. \$562  
 B. \$662  
 C. \$1338   
 D. \$2000

## B. Short Question

In the following exercises, unless otherwise specified, give the answers correct to 2 decimal places if necessary.

8. Complete the following table.

|     | Principal | Interest rate (p.a.) | Period    | Simple interest |
|-----|-----------|----------------------|-----------|-----------------|
| (a) | \$5000    | 3%                   | 4 years   |                 |
| (b) | \$30 000  | 2.5%                 | 1.5 years |                 |
| (c) | \$25 000  | 4%                   | 15 months |                 |

9. Mr. Lee deposits \$18 000 in a bank at a simple interest rate of 3% p.a. Find the amount he will receive after 2 years and 8 months.
10. If the simple interest on \$54 000 at 12% p.a. was \$9720, how long did it deposit?
11. Mr. Lee invests  $\frac{3}{8}$  of his annual salary at a simple interest rate of 12% p.a. If the return of his investment is \$10 800 in a year, what is his monthly salary?
12. If the interest rate is increased from 7% p.a. to 8% p.a., one will get \$200 more for the simple interest in a year. Find the principal.

45. Fanny is a designer. Her net chargeable income is \$120 000. Find her salaries tax payable.
46. Edward's net chargeable income was \$45 000 last year. If his net chargeable income is increased by 16% this year, find the percentage change in his salaries tax payable.
47. Annie pays \$2500 as salaries tax this year. What is her net chargeable income?
48. Tim pays \$6960 as salaries tax this year. What is his net chargeable income?

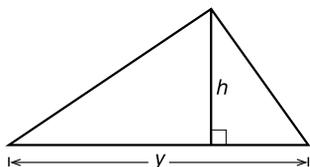


## Advanced Stage

### A. Multiple-choice Question

1. Water and alcohol are mixed in a ratio of 5 : 2. After 2 hours, 20% of alcohol evaporates away. Find the percentage of alcohol left in the mixture correct to 3 significant figures.
- A. 30%  
 B. 24.2%  
 C. 54.5%  
 D. 75.8%
- 
2. If each side of a rectangle is increased by 12%, what is the percentage increase in its area?
- A. 12%  
 B. 24%  
 C. 25.44%  
 D. 44%
- 
3. Room *A* is 10% larger than Room *B* while Room *B* is 10% smaller than Room *C*. Which of the following is true?
- A. Room *C* is more than 1% larger than Room *A*.  
 B. Room *A* is more than 1% larger than Room *C*.  
 C. Room *C* is 1% larger than Room *A*.  
 D. Room *A* has the same size as Room *C*.
- 
4. If a cube is cut into 8 smaller cubes, then the percentage increase in the total surface area is
- A. 50%.  
 B. 100%.  
 C. 150%.  
 D. 20%.
-

5. In the figure, if  $y$  is increased by 25%, what is the percentage decrease in  $h$  such that the area of the triangle remains unchanged?



- A. 15%  
 B. 20%  
 C. 25%  
 D. 30%



6.  $P$  is 60% of  $Q$ . If  $Q$  is increased by 10%, what is the percentage increase in  $P$  such that  $P$  is still 60% of  $Q$ ?
- A. 10%  
 B. 12%  
 C. 14%  
 D. 16%



## B. Long Question

In the following exercises, unless otherwise specified, give the answers correct to 2 decimal places if necessary.

7. The value of a house is \$2 500 000 now. If the value of the house is increased by 8% each year, find
- (a) the value of the house after 3 years;  
 (b) the value of the house 2 years ago.
8. Mr. Chan bought a new car with \$120 000 and the car depreciated by 10% each year. After 3 years, he sold it to Mr. Wong with another 15% discount of its value.
- (a) What was the value of the car after 3 years?  
 (b) How much did Mr. Wong pay for the car?
9. Mr. Wong deposits \$50 000 in a 6-month fixed deposit account at 6% p.a. for 2 years. Find the amount that he will get
- (a) if the interest is taken out at the end of each period,  
 (b) if the interest is not taken out and is compounded with the principal for the next period.



## Public Exam Classroom

### Step-by-Step Demo

If the angle and the radius of a sector are decreased by  $x\%$  and  $75\%$  respectively so that its area is decreased by  $95\%$ , find  $x$ .

- A. 20      B. 40      C. 60      D. 80

**Solution:**

A

Let  $\theta$ ,  $r$  and  $A$  be the original angle, radius and area of the sector respectively.

$$A = \pi r^2 \times \frac{\theta}{360^\circ}$$

$$\text{New area} = \pi[(1 - 75\%)r]^2 \times \frac{(1 - x\%)\theta}{360^\circ}$$

◀ 扇形角減少  $x\%$  及半徑減少  $75\%$ 。

$$= 0.0625(1 - x\%)\pi r^2 \times \frac{\theta}{360^\circ}$$

$$= 0.0625(1 - x\%)A$$

◀ 以  $A$  表示新面積。

$$0.0625(1 - x\%)A = (1 - 95\%)A$$

$$1 - x\% = 0.8$$

$$x = 20$$

#### 考題趨勢

- DSE 卷二常見「成分百分變化 (component percentage change)」及「複利息」題型
- DSE 沒有出現涉及「薪俸稅」的題型

參考 DSE 2020 Paper 2 Q15



### Exam-type Question

- If the base radius and the height of a cylinder are increased by  $20\%$  and  $x\%$  respectively so that its volume is increased by  $80\%$ , find  $x$ .  
A. 15  
B. 25  
C. 75  
D. 85
- A sum of \$25 000 is deposited at an interest rate of  $8\%$  per annum for 1 year, compounded monthly. Find the interest correct to the nearest dollar.  
A. \$2000  
B. \$2070  
C. \$2075  
D. \$4307

## Assessment 1 (Revision for Chapters 1 – 6)

Time allowed: 1 hour and 30 minutes

Full marks: 100

Answer ALL questions

Section A: Multiple-choice Question (30 marks)

Each question carries 2 marks.

1. Simplify  $\frac{(a^{-3}b^2)^4}{a^6b^{-2}}$ .

A.  $\frac{b^4}{a^2}$

B.  $\frac{b^6}{a^6}$

C.  $\frac{b^{10}}{a^{12}}$

D.  $\frac{b^{10}}{a^{18}}$

2.  $13 + 2^4 + 3 \times 2^9 =$

A. 1100011101<sub>(2)</sub>

B. 10000011101<sub>(2)</sub>

C. 11000011101<sub>(2)</sub>

D. 11000101101<sub>(2)</sub>

3. Express  $(4.2 \times 10^{-5}) \div (8.4 \times 10^3)$  in scientific notation.

A.  $2 \times 10^{-8}$

B.  $2 \times 10^{-9}$

C.  $5 \times 10^{-9}$

D.  $5 \times 10^{-10}$

DSE 4. Factorize  $a^2 - b^2 - 4a + 4b$ .

A.  $(a - b)(a + b - 4)$

B.  $(a - b)(a + b + 4)$

C.  $(a + b)(a - b + 4)$

D.  $(a + b)(a - b - 4)$

5. Which of the following CANNOT be factorized?

A.  $x^2 + 8x + 16$

B.  $x^2 - 16$

C.  $x^2 + 6x - 16$

D.  $x^2 - 6x + 16$

6. The value of a car depreciates 12% each year. Find the percentage decrease in the value of the car after it has been used for 3 years.

A. 30%

B. 31.9% (cor. to 3 sig. fig.)

C. 36%

D. 40.5% (cor. to 3 sig. fig.)

7. If the length of a rectangle is increased by 25% and the width is decreased by 20%, then what will be the percentage change in its area?

A. Remain unchanged

B. Increase by 5%

C. Decrease by 5%

D. Increase by 4%

8. Which of the following numbers satisfies the inequality  $\frac{x-2}{3} - \frac{1}{2} > \frac{x}{4}$ ?

A. 14

B. 16

C. -14

D. -16

Section B: Short Question (37 marks)

16. Solve  $3^{x+2} + 3^x = 270$ . (4 marks)

17. (a) Factorize  $x^2 - 4x - 12$ .  
 (b) Hence factorize  $(u - 1)^2 - 4u - 8$ . (5 marks)

**DSE** 18. The price of an iPod is first increased by 20% and then decreased by 15%. What is the overall percentage change in the price? (5 marks)

19. Refer to the following tables of allowances and tax rates.

|                                      |           |
|--------------------------------------|-----------|
| Basic allowance                      | \$132 000 |
| Married person's allowance           | \$264 000 |
| Child allowance (For each dependent) | \$120 000 |

| Net chargeable income | Tax rate (%) |
|-----------------------|--------------|
| First \$50 000        | 2            |
| Next \$50 000         | 6            |
| Next \$50 000         | 10           |
| Next \$50 000         | 14           |
| Remainder             | 17           |

Mr. Mak earned \$24 500 each month last year and his wife earned \$158 000 last year. They have one daughter who is a primary student.

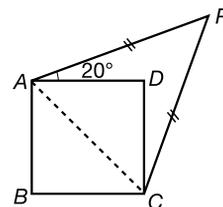
- (a) Find the net chargeable income.  
 (b) Find the salaries tax payable. (5 marks)

20. Solve the inequality  $3 - \frac{x - 2}{4} < \frac{x + 11}{6}$  and represent the solution graphically. (5 marks)

21. In the figure,  $ABCD$  is a square and  $PA = PC$ .

If  $\angle PAD = 20^\circ$ , find

- (a)  $\angle PAC$ ;  
 (b)  $\angle APC$ .



(5 marks)