

1

Measurement and Error



Concept Check

1. Scientific notation

- (a) A very large or very small number can be expressed in scientific notation. It is in the form $a \times 10^n$, where $1 \leq a < 10$ or $-10 < a \leq -1$ and n is an integer.

E.g.

Number	123 456 789.1	-0.000 012 34
Scientific notation	1.23×10^8 (cor. to 3 sig. fig.)	-1.23×10^{-5} (cor. to 3 sig. fig.)

- (b) To convert the number expressed in scientific notation into integers or decimals, just move the decimal points to the left or right by n digits.

E.g. 1.23×10^8 $\xrightarrow{\text{move the decimal point to the right by 8 digits}}$ 1. 2 3 0 0 0 0 0 0 0 i.e. 123 000 000
① ② ③ ④ ⑤ ⑥ ⑦ ⑧

-1.23×10^{-5} $\xrightarrow{\text{move the decimal point to the left by 5 digits}}$ -0 0 0 0 0 1. 2 3 i.e. -0.000 012 3
⑤ ④ ③ ② ①

2. Error

- (a) Absolute error = the difference between the approximated value and the actual value
- (b) Maximum absolute error = half of the scale interval of the measuring tool
- (c) (i) Upper limit = measured value + maximum absolute error
 (ii) Lower limit = measured value – maximum absolute error
 (iii) Lower limit \leq Actual value $<$ Upper limit

(d) Relative error = $\frac{\text{Absolute error}}{\text{Actual value}}$ or $\frac{\text{Maximum absolute error}}{\text{Measured value}}$

Mathematics Tips

Note that relative error and percentage error do not have any units.

(e) Percentage error = Relative error \times 100%

E.g.

For a measurement	
Measured value	13.6 cm
Scale interval	0.1 cm
Maximum absolute error	$\frac{0.1 \text{ cm}}{2} = 0.05 \text{ cm}$
Upper limit	$(13.6 + 0.05) \text{ cm} = 13.65 \text{ cm}$
Lower limit	$(13.6 - 0.05) \text{ cm} = 13.55 \text{ cm}$
Relative error	$\frac{0.05}{13.6} \approx 0.003 \text{ 676}$
Percentage error	$\frac{0.05}{13.6} \times 100\% \approx 0.3676\%$

Approximating an exact value	
Actual value	1234
Approximated value	1230
Absolute error	$1234 - 1230 = 4$
Relative error	$\frac{4}{1234} \approx 0.003 \text{ 241}$
Percentage error	$\frac{4}{1234} \times 100\% \approx 0.3241\%$

Warm-up Section

Matching

Number

1. 100.001 •
2. -0.004 501 •
3. 120 000 •
4. -4501 •
5. 0.000 12 •

Scientific notation

- 1.2×10^{-4}
- 1.2×10^5
- -4.501×10^{-3}
- -4.501×10^3
- $1.000 \text{ 01} \times 10^2$

Fill in the Blanks

6. When 190 is rounded off to 200, find the absolute error and the relative error.

Absolute error = _____ - _____ ◀ absolute error = approximated value - actual value

= _____

Relative error = $\frac{\boxed{}}{\boxed{}}$

◀ relative error = $\frac{\text{absolute error}}{\text{actual value}}$

= $\frac{\boxed{}}{\boxed{}}$

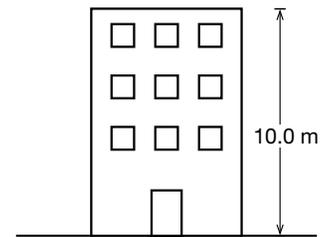
23. The length of Tim's hand span is measured to be 7.8 cm. If the maximum absolute error of the measurement is 0.1 cm, find the percentage error, correct to 3 significant figures.

24. The diameter of a cylinder is about 0.000 065 m. Use scientific notation to represent the radius of the cylinder.

Level 2 Practice

25. When 0.0432 is rounded off to 2 decimal places, find the relative error. ◀ note that we know the exact value in this question

26. The height of a building is 10.0 m (correct to the nearest 0.1 m).
Find the lower and the upper limits of the height of the building.



27. The length of a pen is 10.3 cm (correct to the nearest 0.1 cm). Find the percentage error, correct to 3 significant figures.

28. It is given that a rope is measured to be 10 cm, correct to the nearest cm. Can 10.5 cm be its actual length? Explain your answer.
◀ lower limit \leq actual length $<$ upper limit

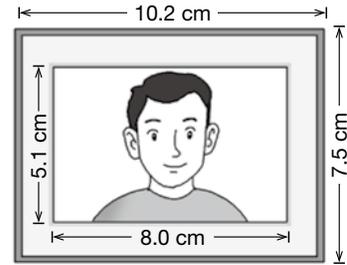
29. The weight of a piece of luggage is measured to be 20 kg. If the relative error is 0.005, find the upper and the lower limits of the actual weight of the luggage.

Challenge Yourself

USE

30. For the cardboard photo frame, the measurements are made by a ruler with a scale interval of 0.1 cm.

- What is the maximum absolute error for the measurements?
- Find the upper limit for the area of the frame. **Scoring Tips 1**



EYA

31. The length of a rope is measured as 4.0 m correct to the nearest 0.1 m.

- Is it possible that the actual length of this rope exceeds 406 cm? Explain your answer.
- Is it possible to cut this rope into 43 pieces of shorter ropes, with each length measured as 10 cm correct to the nearest cm? Explain your answer.

Scoring Tips 1 : Use the upper limits for the outer sides of the frame, and the lower limits for the inner sides of the frame.



Revision Test

Marks: 75

Section A (17 marks, each answer carries 1 mark)

1. Which of the following is NOT an identity?

A. $3(x + 2) = 3x + 6$

B. $2(x - 1)^2 = 2x^2 - 4x + 2$

C. $3(x - 2)(x + 2) = 3x^2 - 4$

D. $(x + 5)(x - 3) = x^2 + 2x - 15$

2. Simplify $\frac{3}{m} - \frac{2}{n}$.

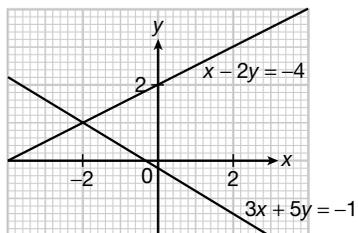
A. $\frac{1}{m - n}$

B. $\frac{3n - 2m}{m - n}$

C. $\frac{1}{mn}$

D. $\frac{3n - 2m}{mn}$

3. Solve the simultaneous equations $\begin{cases} 3x + 5y = -1 \\ x - 2y = -4 \end{cases}$ graphically.



A. $(-2, 1)$

B. $(2, -1)$

C. $(-1, 2)$

D. $(1, -2)$

4. Simplify 3 L : 450 mL.

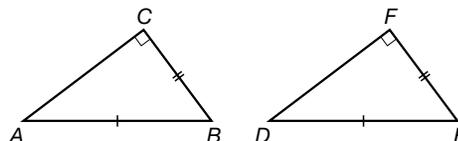
A. 20 : 3

B. 3 : 20

C. 150 : 1

D. 1 : 150

5. What is the condition for $\triangle ABC \cong \triangle DEF$?



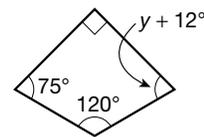
A. SSS

B. SAS

C. AAS

D. RHS

6. In the figure, find y .



A. 51°

B. 63°

C. 75°

D. 90°

7. Rationalize the denominator of $\frac{6}{5\sqrt{3}}$.

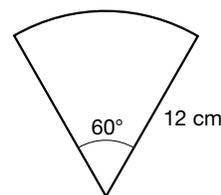
A. $\frac{2\sqrt{3}}{5}$

B. $\frac{5\sqrt{3}}{6}$

C. $\frac{6\sqrt{3}}{5}$

D. $\frac{5\sqrt{3}}{3}$

8. The figure shows a sector. Find the area of the sector in terms of π .

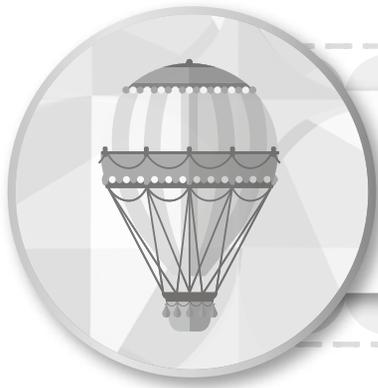


A. $12\pi \text{ cm}^2$

B. $24\pi \text{ cm}^2$

C. $60\pi \text{ cm}^2$

D. $144\pi \text{ cm}^2$



Bridging Task

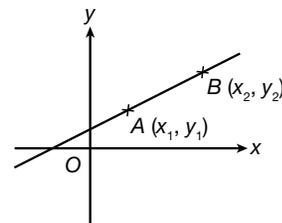
Short Questions

1. It is given that $a^{-m} = \frac{1}{a^m}$. Find the value of $2^{-3} \times 6$.

$2^{-3} \times 6 =$ _____

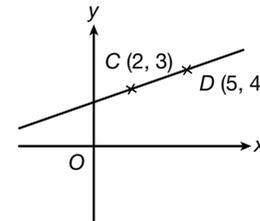
2. In a rectangular coordinate plane, for any two points $A(x_1, y_1)$ and $B(x_2, y_2)$,

the slope of AB is given by $\frac{y_2 - y_1}{x_2 - x_1}$.



In the figure, find the slope of CD .

Slope of $CD =$ _____



3. It is known that the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.

Find the volume of the sphere if $r = 5$ cm. (Give the answer correct to 3 significant figures.)

Volume = _____

4. For a set of data, the mode is the most frequent value of the set of data. For example, the mode of a set of data $\{11, 8, 8, 12, 8, 9\}$ is 8. For the set of data $\{15, 17, 17, 15, 18, 15, 16\}$, find the mode.

Mode = _____