

# 6

## Further Probability 進階概率

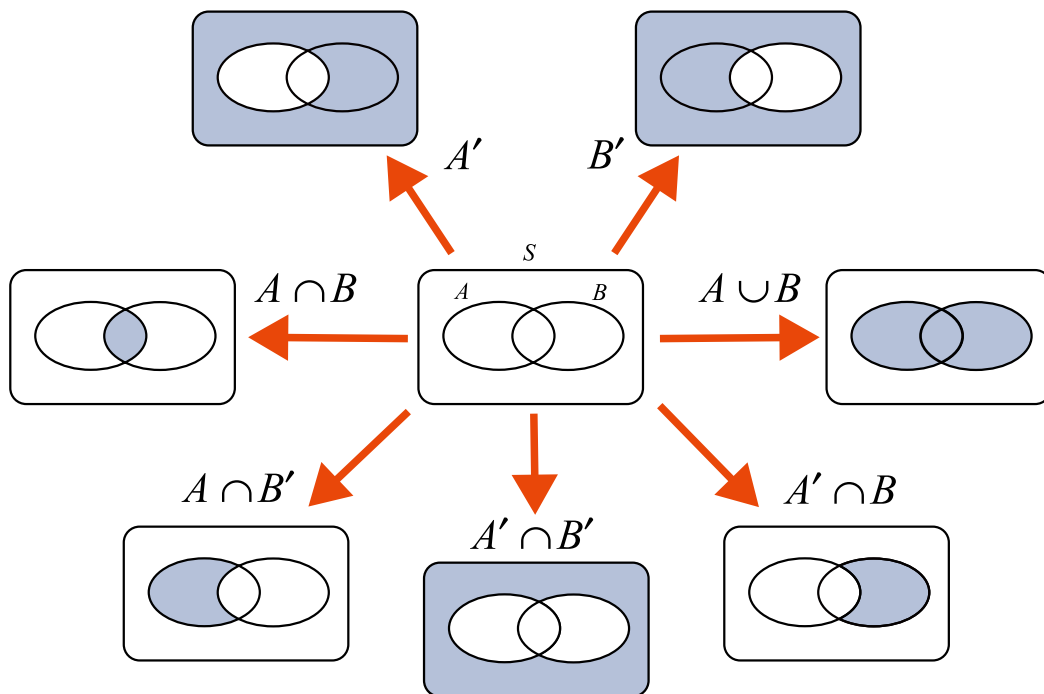
### 速讀筆記

#### 6A Basic Probability 基礎概率

##### (a) Venn Diagram 溫氏圖

Suppose that  $A$  and  $B$  are two events in the sample space  $S$ . Denote the complementary events of  $A$  and  $B$  by  $A'$  and  $B'$  respectively. The following Venn diagrams show some events formed by  $A$  and  $B$ .

假設  $A$  及  $B$  為樣本空間  $S$  內的兩事件。把  $A$  及  $B$  的互補事件分別記為  $A'$  及  $B'$ 。以下溫氏圖展示了一些由  $A$  與  $B$  組成的事件。



#### 應試備忘

- 溫氏圖有助我們理解不同事件之間的關係，從而解決概率問題。

**(b) Properties of Probability 概率的性質**

In a random experiment, if  $A$  is any event in the sample space  $S$ , then

在一隨機實驗中，若  $A$  為樣本空間  $S$  內的任意事件，則

**(i)**  $0 \leq P(A) \leq 1$

**(ii)**  $P(S) = 1$

**(iii)**  $P(\emptyset) = 0$     ◀  $\emptyset$  為 empty set (空集)。

**(iv)**  $P(A') = 1 - P(A)$

**(c) Addition Law of Probability 概率的加法定律**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**(d) Special Terms in Probability 與概率相關的詞彙**

**(i) Independent Events 獨立事件**

$A$  and  $B$  are independent events if and only if  $P(A \cap B) = P(A) \times P(B)$ .

$A$  與  $B$  為獨立事件當且僅當  $P(A \cap B) = P(A) \times P(B)$ 。

**應試備忘**

- 若  $A$  與  $B$  獨立，則  $A$  與  $B'$  獨立、 $A'$  與  $B$  獨立、 $A'$  與  $B'$  也獨立。

**(ii) Mutually Exclusive Events 互斥事件**

$A$  and  $B$  are mutually exclusive events if and only if  $P(A \cap B) = 0$ .

$A$  與  $B$  為互斥事件當且僅當  $P(A \cap B) = 0$ 。

**(iii) Complementary Events 互補事件**

$A$  and  $B$  are complementary events if and only if  $P(A) + P(B) = 1$  and  $P(A \cap B) = 0$ .

$A$  與  $B$  為互補事件當且僅當  $P(A) + P(B) = 1$  及  $P(A \cap B) = 0$ 。

**(iv) Exhaustive Events 窮舉事件**

$A$  and  $B$  are exhaustive events if and only if  $P(A \cup B) = 1$ .

$A$  與  $B$  為窮舉事件當且僅當  $P(A \cup B) = 1$ 。

## 6B Conditional Probability 條件概率

- Suppose that  $P(A) > 0$ . The probability of  $B$  given  $A$  is denoted by  $P(B | A)$ , where 假設  $P(A) > 0$ 。在已知  $A$  發生的情況下  $B$  發生的概率，記為  $P(B | A)$ ，其中

$$P(B | A) = \frac{P(B \cap A)}{P(A)}$$

### 應試備忘

- 已知  $P(A) \neq 0$ 。  $A$  與  $B$  獨立  $\Leftrightarrow P(B) = P(B | A)$ 。

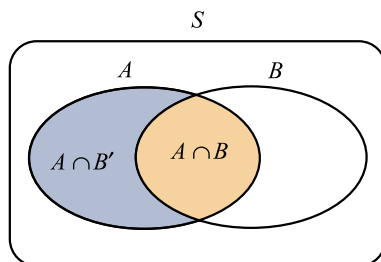
- From the Venn diagram, we have

由溫氏圖，可得

$$P(A \cap B) + P(A \cap B') = P(A)$$

$$\frac{P(A \cap B)}{P(A)} + \frac{P(A \cap B')}{P(A)} = 1$$

$$\therefore P(B | A) + P(B' | A) = 1$$



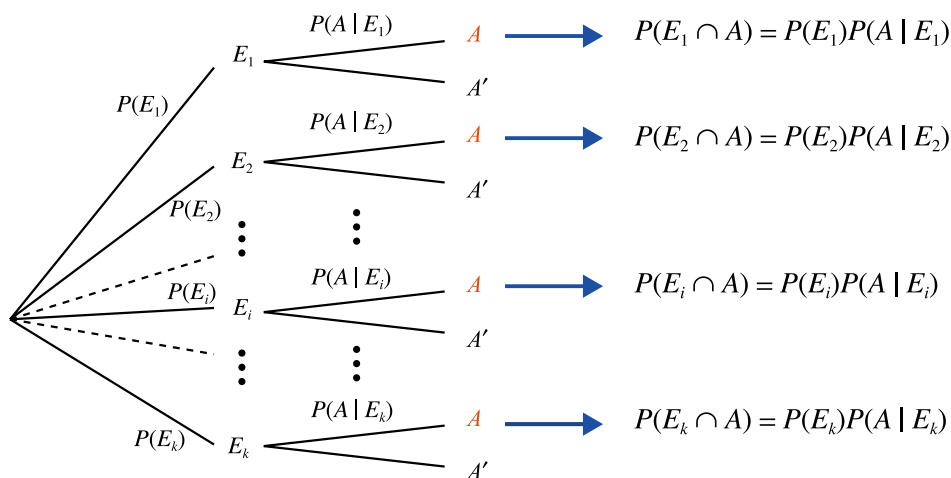
## 6C Bayes' Theorem 貝葉斯定理

In a sample space  $S$ , suppose that  $A$  is any event in  $S$ ,  $E_1, E_2, \dots, E_k$  are  $k$  mutually exclusive and exhaustive events in  $S$ .

在樣本空間  $S$  中，假設  $A$  為  $S$  內任意一個事件， $E_1, E_2, \dots, E_k$  為  $S$  內  $k$  個互斥且窮舉的事件。

### 提示

- (a)  $E_1, E_2, \dots, E_k$  為互斥事件，表示對任意  $i \neq j$ ， $P(E_i \cap E_j) = 0$ 。
- (b)  $E_1, E_2, \dots, E_k$  為窮舉事件，表示  $P(E_1 \cup E_2 \cup \dots \cup E_k) = 1$ 。



- (a) Law of Total Probability 全概率定律

$$P(A) = P(E_1)P(A | E_1) + P(E_2)P(A | E_2) + \dots + P(E_k)P(A | E_k) = \sum_{i=1}^k P(E_i)P(A | E_i)$$

- (b) Bayes' Theorem 貝葉斯定理

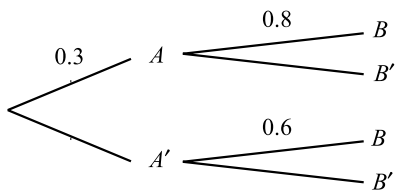
$$P(E_r | A) = \frac{P(E_r \cap A)}{P(A)} = \frac{P(E_r)P(A | E_r)}{P(E_1)P(A | E_1) + P(E_2)P(A | E_2) + \dots + P(E_k)P(A | E_k)}$$

## 應試備忘

- 利用 tree diagram (樹形圖) 有助理解及解決涉及貝葉斯定理的問題。

**e.g. 1.** It is given that  $P(A) = 0.3$ ,  $P(B | A) = 0.8$  and  $P(B | A') = 0.6$ .

已知  $P(A) = 0.3$ 、 $P(B | A) = 0.8$  及  $P(B | A') = 0.6$ 。



$$P(B) = 0.3 \times 0.8 + (1 - 0.3) \times 0.6 \quad \blacktriangleleft \text{ Law of total probability 全概率定律}$$

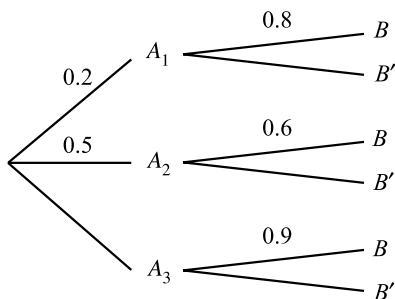
$$= 0.66$$

$$P(A | B) = \frac{0.3 \times 0.8}{0.66} \quad \blacktriangleleft \text{ Bayes' theorem 貝葉斯定理}$$

$$= \frac{4}{11}$$

**2.** It is given that  $A_1, A_2$  and  $A_3$  are mutually exclusive events such that  $A_1 \cup A_2 \cup A_3 = S$ ,  $P(A_1) = 0.2$ ,  $P(A_2) = 0.5$ ,  $P(B | A_1) = 0.8$ ,  $P(B | A_2) = 0.6$  and  $P(B | A_3) = 0.9$ .

已知  $A_1$ 、 $A_2$  及  $A_3$  為互斥事件使得  $A_1 \cup A_2 \cup A_3 = S$ 、 $P(A_1) = 0.2$ 、 $P(A_2) = 0.5$ 、 $P(B | A_1) = 0.8$ 、 $P(B | A_2) = 0.6$  及  $P(B | A_3) = 0.9$ 。



$$P(B) = 0.2 \times 0.8 + 0.5 \times 0.6 + (1 - 0.2 - 0.5) \times 0.9 \quad \blacktriangleleft \text{ Law of total probability 全概率定律}$$

$$= 0.73$$

$$P(A_2 | B) = \frac{0.5 \times 0.6}{0.73} \quad \blacktriangleleft \text{ Bayes' theorem 貝葉斯定理}$$

$$= \frac{30}{73}$$

Use Venn Diagram to find probability

利用溫氏圖求概率

注意

- 當題目涉及不同事件之間的關係，使用 Venn diagram (溫氏圖) 有助我們理解這些關係。
- 小心分辨 mutually exclusive events (互斥事件) 與 independent events (獨立事件)。

應試例題6.1

Let  $X$  and  $Y$  be two events such that  $P(X) = 0.35$ ,  $P(Y) = 0.55$  and  $P(X \cup Y) = 0.85$ .

設  $X$  及  $Y$  為兩事件使得  $P(X) = 0.35$ 、 $P(Y) = 0.55$  及  $P(X \cup Y) = 0.85$ 。

(a) Are  $X$  and  $Y$  mutually exclusive? Explain your answer.

$X$  與  $Y$  是否互斥? 解釋你的答案。

(b) Are  $X$  and  $Y$  independent? Explain your answer.

$X$  與  $Y$  是否獨立? 解釋你的答案。

(c) Find  $P(X' \cap Y)$ .

求  $P(X' \cap Y)$ 。

(7 marks)

題解

(a) 分析

$X$  與  $Y$  為 mutually exclusive (互斥)  $\Leftrightarrow P(X \cap Y) = 0$ 。

因此，我們先計算  $P(X \cap Y)$ 。

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$0.85 = 0.35 + 0.55 - P(X \cap Y)$$

$$P(X \cap Y) = 0.05$$

1A

$$\therefore P(X \cap Y) \neq 0$$

1M

$\therefore X$  and  $Y$  are not mutually exclusive.

1A

$X$  與  $Y$  不是互斥。

提示

Addition Law of Probability

概率的加法定律

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) 分析

$X$  與  $Y$  為 independent (獨立)  $\Leftrightarrow P(X) \times P(Y) = P(X \cap Y)$

留意，我們已在 (a) 求得  $P(X \cap Y)$ 。

$$\begin{aligned} P(X) \times P(Y) \\ &= 0.35 \times 0.55 \\ &= 0.1925 \end{aligned}$$

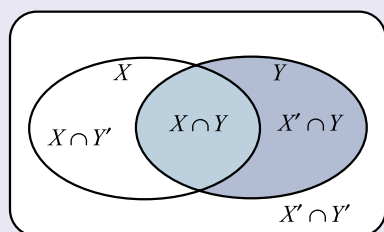
$$\neq P(X \cap Y) \quad \text{由 (a), } P(X \cap Y) = 0.05 \quad \mathbf{1M}$$

$\therefore X$  and  $Y$  are not independent.  $\mathbf{1A}$

$X$  及  $Y$  不是獨立。

(c) 分析

以下 Venn diagram (溫氏圖) 有助我們理解所求概率。



$$\begin{aligned} P(X' \cap Y) \\ &= P(Y) - P(X \cap Y) \\ &= 0.55 - 0.05 \quad \mathbf{1M} \\ &= \underline{0.5} \quad \mathbf{1A} \end{aligned}$$

應試備忘

- 別混淆 mutually exclusive (互斥) 與 independent (獨立)，公開試中不少考生常把兩者混淆。
- 若  $X$  與  $Y$  獨立，則  $X'$  與  $Y$  也獨立，可得  $P(X' \cap Y) = P(X') \times P(Y)$ 。  
不過，本例題中  $X$  與  $Y$  不是獨立，所以 (c) 不可運用以上關係。  
本例題 (c) 所展示的方法，無論  $X$  與  $Y$  是否獨立也適用。
- 同一事件的概率，可用不同形式表示，例如在 (c) 中的  $P(X' \cap Y)$ ，可表示為  
**(I)**  $P(Y) - P(X \cap Y)$   
**(II)**  $P(X') - P(X' \cap Y)$   
我們應視乎已有資料來選用合適形式來計算。

應試訓練 6

1. Let  $A$  and  $B$  be two independent events such that  $P(A \cap B) = 0.22$ ,  $P(A \cup B) = 0.73$  and  $P(A) > P(B)$ .
- (a) Find  $P(A)$ .
  - (b) Find  $P(A \cap B')$ .

(6 marks)

設  $A$  及  $B$  為兩獨立事件使得  $P(A \cap B) = 0.22$ 、 $P(A \cup B) = 0.73$  及  $P(A) > P(B)$ 。

- (a) 求  $P(A)$ 。
- (b) 求  $P(A \cap B')$ 。

(6 分)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

1

2

3

B

4

5

B

6

7

B

8

B

A

2. Let  $X$  and  $Y$  be two events such that  $P(X \cap Y) = 0.52$  and  $P(X \cap Y') = 0.24$ , where  $Y'$  denotes the complementary event of  $Y$ .

(a) Find  $P(X)$ .

(b) It is given that  $X$  and  $Y$  are independent.

(i) Find  $P(Y)$ .

(ii) Find  $P(X' \cap Y')$ .

(7 marks)

設  $X$  及  $Y$  為兩事件使得  $P(X \cap Y) = 0.52$  及  $P(X \cap Y') = 0.24$ ，其中  $Y'$  表示事件  $Y$  的互補事件。

(a) 求  $P(X)$ 。

(b) 已知  $X$  與  $Y$  獨立。

(i) 求  $P(Y)$ 。

(ii) 求  $P(X' \cap Y')$ 。

(7 分)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Use Venn Diagram to find conditional probability

利用溫氏圖求條件概率

注意

• 以集合表示事件，然後利用溫氏圖有助解涉及條件概率的問題。

•  $P(B | A) = \frac{P(B \cap A)}{P(A)}$

應試例題6.2

參考：2014Q08

In a club,  $\frac{1}{5}$  of the members are smokers. It is known that  $\frac{1}{6}$  of the male members are smokers.

Among the non-smokers,  $\frac{1}{8}$  of them are females. Suppose a member is randomly selected.

在某俱樂部中， $\frac{1}{5}$  的會員為吸煙者。已知  $\frac{1}{6}$  的男性會員為吸煙者。在非吸煙者中， $\frac{1}{8}$  為女性。假設隨機選出一名會員。

- (a) What is the probability that the selected member is a female non-smoker?  
選出的會員為女性非吸煙者的概率是多少？
- (b) What is the probability that the selected member is a male?  
選出的會員為男性的概率是多少？
- (c) If the selected member is a female, what is the probability that she is a non-smoker?  
若選出的會員為女性，她是非吸煙者的概率是多少？

(6 marks)

題解

分析

首先以集合表示各事件，再將題目所有資料寫出並化簡，之後再以 Venn diagram (溫氏圖) 表示。

Let  $M$  be the event of selecting a male,  $X$  be the event of selecting a smoker. According to the question, we have

設  $M$  為選出一名男性會員的事件， $X$  為選出一名吸煙者的事件。依題意，可得

$P(X) = \frac{1}{5}$

$P(X | M) = \frac{1}{6}$

$P(M' | X') = \frac{1}{8}$

$\frac{P(X \cap M)}{P(M)} = \frac{1}{6}$

$\frac{P(M' \cap X')}{P(X')} = \frac{1}{8}$

$P(X \cap M) = \frac{1}{6}P(M) \dots \dots \dots (1)$

$P(M' \cap X') = \frac{1}{8}P(X') \dots \dots \dots (2)$

(a) The required probability

所求概率

$$\begin{aligned}
 &= P(M' \cap X') \\
 &= \frac{1}{8} P(X') \quad (\text{by (2)}) \\
 &= \frac{1}{8} [1 - P(X)] \\
 &= \frac{1}{8} \times \left(1 - \frac{1}{5}\right) \\
 &= \frac{1}{10}
 \end{aligned}$$

1A

(b) Note that the required probability is  $P(M)$ .

留意所求概率為  $P(M)$ 。

$$P(X \cap M) = \frac{1}{6} P(M) \quad (\text{by (1)})$$

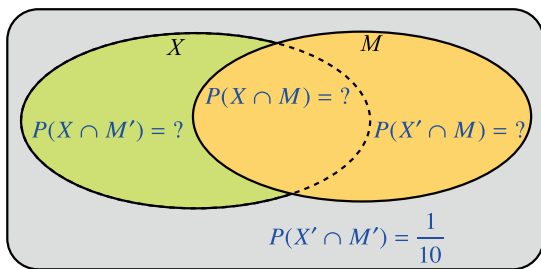
$$P(X' \cap M) = P(M) - P(X \cap M) = \frac{5}{6} P(M)$$

$$P(X \cap M') = P(X) - P(X \cap M) = \frac{1}{5} - \frac{1}{6} P(M)$$

$$\therefore P(M) + \left(\frac{1}{5} - \frac{1}{6} P(M)\right) + \frac{1}{10} = 1$$

$$\frac{5}{6} P(M) = \frac{7}{10}$$

$$P(M) = \frac{21}{25}$$



1M + 1M

$$P(M) + P(X \cap M') + P(X' \cap M) = 1$$

1A

(c) The required probability

所求概率

$$\begin{aligned}
 &= P(X' \mid M') \\
 &= \frac{P(X' \cap M')}{P(M')} \\
 &= \frac{\frac{1}{10}}{1 - \frac{21}{25}} \\
 &= \frac{5}{8}
 \end{aligned}$$

1M

1A

### 應試訓練 6

3. Let  $A$  and  $B$  be two events. Suppose that  $P(A) = 0.6$ ,  $P(B') = 0.8$  and  $P(A' | B') = 0.5$ , where  $A'$  and  $B'$  are the complementary events of  $A$  and  $B$  respectively.
- (a) Find  $P(A' \cap B')$  and  $P(A \cap B')$ .
- (b) Are  $A$  and  $B$  mutually exclusive? Explain your answer.

(6 marks)

設  $A$  及  $B$  為兩事件。假定  $P(A) = 0.6$ 、 $P(B') = 0.8$  及  $P(A' | B') = 0.5$ ，其中  $A'$  及  $B'$  分別為  $A$  及  $B$  的互補事件。

- (a) 求  $P(A' \cap B')$  及  $P(A \cap B')$ 。
- (b)  $A$  與  $B$  是否互斥？解釋你的答案。

(6 分)

參考：2015Q02

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

4. Let  $A$  and  $B$  be two events. Denote the complementary event of  $A$  by  $A'$ .

It is given that  $P(B | A') = 0.45$ ,  $P(B) = 0.7$  and  $P(A') = k$ , where  $k > 0$ .

(a) Find  $P(A \cap B)$  in terms of  $k$ .

(b) Suppose that  $P(A \cup B) = 0.95$ .

(i) Find the value of  $k$ .

(ii) Determine where  $A$  and  $B$  are independent. Explain your answer.

(6 marks)

設  $A$  及  $B$  為兩事件。將  $A$  的互補事件記為  $A'$ 。

已知  $P(B | A') = 0.45$ 、 $P(B) = 0.7$  及  $P(A') = k$ ，其中  $k > 0$ 。

(a) 以  $k$  表  $P(A \cap B)$ 。

(b) 假定  $P(A \cup B) = 0.95$ 。

(i) 求  $k$  的值。

(ii) 判別  $A$  與  $B$  是否獨立。解釋你的答案。

(6 分)

參考：2014Q07

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

5. Before a long holiday, the probability for Ivy to have a haircut is 0.35, while the probability for her to have a facial treatment is 0.6. If she has a facial treatment before a long holiday, the probability for her to have a haircut before the holiday is 0.42.

- (a) Given that Ivy has a haircut before a certain long holiday, find the probability that she does not have a facial treatment before that long holiday.
- (b) Given that Ivy does not have a facial treatment before a certain long holiday, find the probability that she has a haircut before that long holiday.
- (c) Find the probability of that, before a certain long holiday, Ivy has neither a haircut nor a facial treatment.

(6 marks)

在某個長假期前，雅琳會去理髮的概率為 0.35，而她会去做面部護理的概率為 0.6。若她在某長假期前去了做面部護理，她會在該長假期前去理髮的概率為 0.42。

- (a) 已知雅琳在某個長假期前去理髮，求她在該長假期前沒有去做面部護理的概率。
- (b) 已知雅琳在某個長假期前沒有去做面部護理，求她在該長假期前去理髮的概率。
- (c) 求在某個長假期前，雅琳既沒有去理髮，也沒有去做面部護理的概率。

(6 分)

參考：2014Q08

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

## Find the probability by Bayes' theorem

利用貝葉斯定理求概率

## 注意

- 利用 tree diagram (樹形圖) 有理解及解決涉及貝葉斯定理的問題。

## 應試例題6.3

A factory manufactures electronic parts by three production lines  $A$ ,  $B$  and  $C$  in the ratio  $1 : 3 : 4$ . It is known that  $4\%$ ,  $5\%$  and  $2\%$  of the electronic parts from  $A$ ,  $B$  and  $C$  are defective respectively.

某工廠按  $1 : 3 : 4$  的比例由生產線  $A$ 、 $B$  及  $C$  生產電子零件。已知生產線  $A$ 、 $B$  及  $C$  中分別有  $4\%$ 、 $5\%$  及  $2\%$  的電子零件有缺陷。

(a) An electronic part is randomly chosen.

隨機抽出一個電子零件。

(i) Find the probability that the part is defective.

求該電子零件有缺陷的概率。

(ii) Find the probability that the part is from production line  $C$  given that it is defective.

已知該電子零件有缺陷，求該電子零件由生產線  $C$  所生產的概率。

(b) If two electronic parts are randomly chosen, find the probability that both are from the same production line and defective.

若隨機抽出兩個電子零件，求它們均來自同一生產線且均有缺陷的概率。

(6 marks)

←————— 題解 —————→

Let  $A$ ,  $B$  and  $C$  be the events of choosing an electronic part of from production lines  $A$ ,  $B$  and  $C$  respectively. Let  $D$  be the event of choosing a defective part.

設  $A$ 、 $B$  及  $C$  分別為抽出一個由生產線  $A$ 、 $B$  及  $C$  所生產的電子零件的事件。設  $D$  為抽出一個有缺陷的電子零件的事件。

(a) (i) The required probability

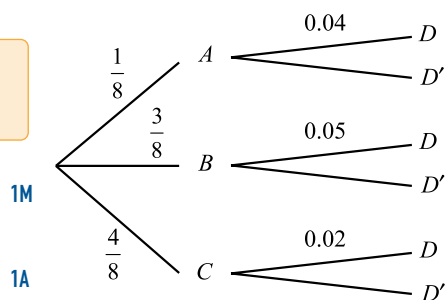
所求概率

$$= P(D)$$

$$= \frac{1}{8} \times 0.04 + \frac{3}{8} \times 0.05 + \frac{4}{8} \times 0.02$$

$$= \frac{27}{800}$$

Law of total probability  
全概率定律



(ii) The required probability

所求概率

$$= P(C \mid D)$$

$$= \frac{P(C \cap D)}{P(D)}$$

$$= \frac{P(C) \times P(D \mid C)}{P(D)}$$

$$= \frac{\frac{4}{8} \times 0.02}{\frac{27}{800}}$$

$$= \frac{8}{27} \text{ (r.t. 0.2963)}$$

(b) The required probability

所求概率

$$= \left(\frac{1}{8} \times 0.04\right)^2 + \left(\frac{3}{8} \times 0.05\right)^2 + \left(\frac{4}{8} \times 0.02\right)^2$$

$$= \frac{61}{128000} \text{ (r.t. 0.0005)}$$

1

2

3

B

4

5

B

6

7

B

8

B

A

### 應試訓練 6

6. Suppose that Anson either leaves on time or leaves late after his dance training. If Anson leaves on time after his dance training on a certain day, the probability for him to leave on time on the next day is 0.22. If he leaves late on a certain day, the probability for him to leave on time on the next day is  $p$ . It is known that Anson leaves late after his dance training on Thursday, and the probability for him to leave on time on both Friday and Saturday is 0.165.
- (a) Find  $p$ .
  - (b) Suppose that Anson will leave late on Saturday, find the probability that he will leave late on Friday.
  - (c) Suppose that Anson will leave on time on Saturday, find the probability that he will leave late on Friday.

(6 marks)

假定瀚霆在舞蹈訓練後必會準時離開或延遲離開。若瀚霆在某一天舞蹈訓練後準時離開，則他在隨後的一天準時離開的概率為 0.22。若他在某一天舞蹈訓練後延遲離開，則他在隨後的一天準時離開的概率為  $p$ 。已知瀚霆在星期四舞蹈訓練後延遲離開，而他在星期五及星期六均準時離開的概率為 0.165。

- (a) 求  $p$ 。
- (b) 假設瀚霆在星期六延遲離開，求他在星期五延遲離開的概率。
- (c) 假設瀚霆在星期六準時離開，求他在星期五延遲離開的概率

(6 分)

---

---

---

---

---

---

---

---

---

---

---



7. In a survey, an interviewee will be asked to choose 1 out of 5 envelopes randomly. Each envelope contains a statement. Among the statements, 3 of them are 'I am a drug abuser' and the others are 'I am NOT a drug abuser'. The interviewee considers the statement and answers either 'True' or 'False' honestly. Let  $p$  be the probability that a randomly selected interviewee is a drug abuser. In the survey, 76 out of 180 interviewees answer 'True'.

- (a) Find  $p$ .
- (b) Given that a randomly selected interviewee answers 'True', find the probability that the interviewee chooses an envelope with the statement 'I am NOT a drug abuser'.

(6 marks)

在一項調查中，一受訪者須在 5 個信封中隨機抽出 1 個。每個信封內有一張字條，其中 3 張寫上了「我是吸毒者」，其餘的則寫上了「我不是吸毒者」。該受訪者考慮字條上的字句後須誠實地回答「正確」或「錯誤」。設  $p$  為一隨機選出的受訪者為吸毒者的概率。在該調查中，180 名受訪者中有 76 名回答「正確」。

- (a) 求  $p$ 。
- (b) 已知一隨機選出的受訪者回答「正確」，求該受訪者抽出的信封內有一張寫上了「我不是吸毒者」的字條的概率。

(6 分)

參考：2015Q03

---



---



---



---



---



---



---



---



---



---



---

## 甲部 應試特訓 6

Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

除特別指明外，數值答案須用真確值或四位小數表示。

### 應試技巧 6.1

1. Suppose that  $A$  and  $B$  are two events such that  $P(A) = 0.6$  and  $P(B) = 0.9$ . Let  $P(A \cap B) = k$ .
- Express  $P(A \cup B)$  in terms of  $k$ .
  - Prove that  $0.5 \leq k \leq 1$ .
  - Denote the complementary event of  $A$  by  $A'$ . Prove that  $0 \leq P(A' \cap B) \leq 0.4$ .

(7 marks)

假定  $A$  及  $B$  為兩事件使得  $P(A) = 0.6$  及  $P(B) = 0.9$ 。設  $P(A \cap B) = k$ 。

- 以  $k$  表  $P(A \cup B)$ 。
- 證明  $0.5 \leq k \leq 1$ 。
- 將  $A$  的互補事件記為  $A'$ 。證明  $0 \leq P(A' \cap B) \leq 0.4$ 。

(7 分)

### 應試技巧 6.2

2. Let  $A$  and  $B$  be two events such that  $P(B | A) = \frac{1}{2}$ ,  $P(A | B') = \frac{4}{7}$  and  $P(A) = a$ .
- Express  $P(A \cap B)$  and  $P(B)$  in terms of  $a$ .
  - If  $P(A' \cap B') = 0.15$ , find  $a$ .

(6 marks)

設  $A$  及  $B$  為兩事件使得  $P(B | A) = \frac{1}{2}$ 、 $P(A | B') = \frac{4}{7}$  及  $P(A) = a$ 。

- 以  $a$  表  $P(A \cap B)$  及  $P(B)$ 。
- 若  $P(A' \cap B') = 0.15$ ，求  $a$ 。

(6 分)

3. Let  $A$  and  $B$  be two events. Denote the complementary events of  $A$  and  $B$  by  $A'$  and  $B'$  respectively. It is given that  $P(A | B') = \frac{3}{8}$  and  $P(B) = \frac{2}{5}$ .
- (a) Find  $P(A \cup B)$ .
- (b) It is given that  $A$  and  $B$  are independent.
- (i) Find  $P(A)$ .
- (ii) Determine whether  $A'$  and  $B$  are mutually exclusive. Explain your answer.

(6 marks)

設  $A$  及  $B$  為兩事件。將  $A$  及  $B$  的互補事件分別記為  $A'$  及  $B'$ 。

已知  $P(A | B') = \frac{3}{8}$  及  $P(B) = \frac{2}{5}$ 。

- (a) 求  $P(A \cup B)$ 。
- (b) 已知  $A$  與  $B$  獨立。
- (i) 求  $P(A)$ 。
- (ii) 判別  $A'$  與  $B$  是否互斥。解釋你的答案。

(6 分)

參考：2023Q04

4. Let  $A$  and  $B$  be two events. Denote the complementary event of  $B$  by  $B'$ . It is given that  $P(A | B') = 4P(A | B)$ . Suppose that  $P(A \cap B) = 0.4$  and  $P(B) = b$ , where  $b \neq 0$ .
- (a) Prove that  $P(A) = \frac{1.6}{b} - 1.2$ .
- (b) Let  $C$  be an event such that  $P(C) = 0.65$ . Are  $A$  and  $C$  mutually exclusive? Explain your answer.
- (c) If  $P(B) = 0.8$ , are  $A$  and  $B$  independent? Explain your answer.

(7 marks)

設  $A$  及  $B$  為兩事件。將  $B$  的互補事件記為  $B'$ 。已知  $P(A | B') = 4P(A | B)$ 。假定  $P(A \cap B) = 0.4$  及  $P(B) = b$ ，其中  $b \neq 0$ 。

- (a) 證明  $P(A) = \frac{1.6}{b} - 1.2$ 。
- (b) 設  $C$  為一事件使得  $P(C) = 0.65$ 。  $A$  與  $C$  是否互斥？解釋你的答案。
- (c) 若  $P(B) = 0.8$ ，  $A$  與  $B$  是否獨立？解釋你的答案。

(7 分)

參考：2022Q03

5. In a school, 72% of the students wear glasses, where 60% of them are boys. 25% of the girls do not wear glasses. A student in this school is selected at random.
- (a) Find the probability that the selected student is a boy who wears glasses.
  - (b) Find the probability that the selected student is a girl.
  - (c) Find the probability that the selected student does not wear glasses given that he is a boy.

(7 marks)

在一所學校中，72% 的學生戴眼鏡，其中 60% 為男生。25% 的女生不戴眼鏡。從該學校隨機選出一名學生。

- (a) 求所選出的學生為戴眼鏡的男生的概率。
- (b) 求所選出的學生為女生的概率。
- (c) 已知所選出的是男生，求該學生不戴眼鏡的概率。

(7 分)

參考：2014Q08

6. In a supermarket, a customer can only pay the bill through the cashier or the automatic machine. In either case, the customer can only choose to pay by cash or by electronic means. The probability for a customer to pay the bill through the automatic machine is  $\frac{1}{5}$ . Given that a customer pays the bill through the cashier, the probability for the customer to pay by electronic means is  $\frac{4}{9}$ .
- (a) Find the probability that a customer pays the bill through the cashier and by electronic means.
  - (b) Given that a customer pays the bill by cash, the probability for the customer to pay through the automatic machine is  $\frac{13}{45}$ .
    - (i) Find the probability that a customer pays by electronic means.
    - (ii) Given that a customer pays by electronic means, find the probability that the customer pays through the automatic machine.

(7 marks)

在某超級市場，顧客只可以到收銀台或自助收銀機付款。無論用哪方式付款，都只可以現金或電子方式支付款項。一顧客選用自助收銀機的概率為  $\frac{1}{5}$ 。已知一顧客到收銀台付款，該顧客以電子方式支付款項的概率為  $\frac{4}{9}$ 。

(a) 求一顧客到收銀台且用電子方式付款的概率。

(b) 已知一顧客以現金付款，該顧客到自助收銀機付款的概率為  $\frac{13}{45}$ 。

(i) 求一顧客以電子方式付款的概率。

(ii) 已知一顧客以電子方式付款，求該顧客到自助收銀機付款的概率。

(7 分)

參考：2023Q03

### 應試技巧 6.3

7. It is given that 2% of youths living in a facility are users of a psychoactive drug. A test is designed to detect whether a youth is a user of this drug. If a youth is a user of this drug, then the probability of the test returning a positive result is 98%. If a youth is not a user of this drug, then the probability of the test returning a negative result is 99%.

(a) Find the probability that a test gives a positive result.

(b) Find the probability that a test gives a false result.

(c) Given that a test gives a negative result, is the probability that the test gives a false result greater than 0.0005? Explain your answer.

(6 marks)

某設施內，已知 2% 的青少年為某特定精神藥物的使用者。一測試能檢測青少年是否為該藥物的使用者。若某青少年為該藥物的使用者，則該測試結果為陽性的概率為 98%。若某青少年不是該藥物的使用者，則該測試結果為陰性的概率為 99%。

(a) 求某次測試的結果為陽性的概率。

(b) 求某次測試的結果錯誤的概率。

(c) 已知某次測試的結果為陰性，該測試的結果錯誤的概率是否大於 0.0005？解釋你的答案。

(6 分)

參考：2021Q02

8. In a factory, smartphones are produced by production lines  $X$  and  $Y$  only. It is known that 1.2% of smartphones from production line  $X$  are defective and 0.8% of smartphones from production line  $Y$  are defective. As a result, 0.92% of smartphones produced by the factory are defective. A smartphone produced by the factory is randomly chosen.
- (a) Find the probability that the chosen smartphone is produced by production line  $X$ .
  - (b) Find the probability that the chosen smartphone is produced by production line  $Y$  and is non-defective.
  - (c) Given that the chosen smartphone is non-defective, find the probability that it is produced by production line  $X$ .

(6 marks)

在某工廠，智能手機只由生產線  $X$  及生產線  $Y$  所生產。已知生產線  $X$  所生產的智能手機有 1.2% 有缺陷，而生產線  $Y$  所生產的智能手機則有 0.8% 有缺陷。結果，該工廠所生產的智能手機有 0.92% 有缺陷。隨機抽出一部由該工廠生產的智能手機。

- (a) 求所抽出的智能手機是由生產線  $X$  所生產的概率。
- (b) 求所抽出的智能手機是由生產線  $Y$  所生產且沒有缺陷的概率。
- (c) 已知所抽出的智能手機沒有缺陷，求該智能手機是由生產線  $X$  所生產的概率。

(6 分)

# Section B Train-up Zone (I)

## 乙部訓練場 (I)

### 應試例題 1

參考：2018Q12

An ecologist studied the spread of a certain virus on pigs in a shed. The temperature  $T$  (in  $^{\circ}\text{C}$ ) in the shed can be modelled by the following expression:

$$ae^T = 5^{bt},$$

where  $a$  and  $b$  are constants and  $t$  ( $0 \leq t \leq 24$ ) is the number of hours elapsed since the study begin. It is given that the slope and the intercept on the vertical axis of the linear function of  $T$  against  $t$  are  $\ln(5^{-0.2})$  and  $\ln 10$  respectively.

一位生態學家研究某病毒在豬棚內的散播情況。豬棚內的溫度  $T$  (以  $^{\circ}\text{C}$  為單位) 可用下式模擬：

$$ae^T = 5^{bt},$$

其中， $a$  及  $b$  均為常數且  $t$  ( $0 \leq t \leq 24$ ) 為研究開始起計所經過的時數。已知對  $T$  與  $t$  的線性函數的圖像的斜率及垂直軸上的截距分別為  $\ln(5^{-0.2})$  及  $\ln 10$ 。

- (a) Express  $T$  in terms of  $t$  and hence find  $a$  and  $b$ .

應試技巧 1.2

以  $t$  表  $T$ 。由此，求  $a$  及  $b$ 。

(2 marks)

- (b) It is given that  $T = \ln\left(\frac{20 - 2R}{R}\right)$ , where  $R$  be the number of pigs being infected by the virus.

已知  $T = \ln\left(\frac{20 - 2R}{R}\right)$ ，其中  $R$  為受感染的豬隻數目。

- (i) Prove that  $R = \frac{10}{5^{1-0.2t} + 1}$ .

證明  $R = \frac{10}{5^{1-0.2t} + 1}$ 。

- (ii) Find  $\frac{dR}{dt}$  and  $\frac{d^2R}{dt^2}$ .

應試技巧 2.1

求  $\frac{dR}{dt}$  及  $\frac{d^2R}{dt^2}$ 。

- (iii) Describe how  $\frac{dR}{dt}$  varies during the study. Explain your answer.

應試技巧 3.4

描述  $\frac{dR}{dt}$  在研究期間如何變化。試解釋你的答案。

(9 marks)

← 題解 →

(a)  $\ln(ae^T) = \ln(5^{bt})$

左方及右方同時使用  $\ln$ 。

$$\ln a + \ln e^T = (\ln 5)bt$$

$$T = \frac{(\ln 5)bt - \ln a}{b}$$

1A

由對數性質，可得  $\ln(5^{bt}) = bt \ln 5$ ，也可寫成  $(\ln 5)bt$ ，但千萬不要寫成  $\ln 5 bt$ 。

$$\begin{cases} (\ln 5)b = \ln 5^{-0.2} \\ -\ln a = \ln 10 \end{cases}$$

$$\Rightarrow \begin{cases} b = -0.2 \\ a^{-1} = 10 \end{cases}$$

表達成線性關係 (linear relationship)

$$\Rightarrow \begin{cases} b = \underline{\underline{-0.2}} \\ a = \underline{\underline{0.1}} \end{cases}$$

1A

(b) (i) 分析

由 (a)，可以  $t$  表示  $T$ ，代入 (b) 所給的數式，便可得一個連繫  $R$  與  $t$  的方程，從而證明所需的數式。

$$T = (\ln 5)(-0.2t) - \ln 0.1 = \ln\left(\frac{5^{-0.2t}}{0.1}\right) = \ln[10(5^{-0.2t})]$$

以  $T$  為主項，以便於 (b)(i) 中建立  $R$  與  $t$  的方程。

$$\ln[10(5^{-0.2t})] = \ln\left(\frac{20 - 2R}{R}\right)$$

$$10(5^{-0.2t}) = \frac{20 - 2R}{R}$$

1M

$$10(5^{-0.2t})R = 20 - 2R$$

$$[10(5^{-0.2t}) + 2]R = 20$$

$$R = \frac{20}{10(5^{-0.2t}) + 2}$$

$$= \frac{10}{5(5^{-0.2t}) + 1}$$

$$= \frac{10}{5^{1-0.2t} + 1}$$

1

(ii)  $\frac{dR}{dt} = -\frac{10(5^{1-0.2t})(\ln 5)(-0.2)}{(5^{1-0.2t} + 1)^2}$

1M

$$= \frac{2(\ln 5)5^{1-0.2t}}{(5^{1-0.2t} + 1)^2}$$

1A

提示

- $\frac{d}{dx}[g(x)]^{-1} = -[g(x)]^{-2} \frac{d}{dx}[g(x)]$
- $\frac{d}{dx}(a^{kx}) = a^{kx}(\ln a)(k)$



$$\begin{aligned} & \frac{d^2R}{dt^2} \\ &= \frac{(5^{1-0.2t} + 1)^2 \cdot 2(\ln 5)(-0.2 \ln 5)5^{1-0.2t} - 2(\ln 5)5^{1-0.2t} \cdot 2(5^{1-0.2t} + 1)(-0.2 \ln 5)5^{1-0.2t}}{(5^{1-0.2t} + 1)^4} \\ &= \frac{-0.4(\ln 5)^2 5^{1-0.2t} [5^{1-0.2t} + 1 - 2(5^{1-0.2t})]}{(5^{1-0.2t} + 1)^3} \quad \text{1M (for quotient rule)} \\ &= \frac{0.4(\ln 5)^2 5^{1-0.2t} (5^{1-0.2t} - 1)}{(5^{1-0.2t} + 1)^3} \quad \text{1A} \end{aligned}$$

消去分子及分母的公因式  $(5^{1-0.2t} + 1)$ ，然後提取分子的公因式  $-0.4(\ln 5)^2 5^{1-0.2t}$ 。

(iii) 分析

因現正考慮  $\frac{dR}{dt}$  的變化，所以須計算  $\frac{d^2R}{dt^2}$ ，為  $\frac{dR}{dt}$  進行「1st derivative test (一階導數判別法)」。

$$\begin{aligned} \frac{d^2R}{dt^2} &= 0 \\ 5^{1-0.2t} &= 1 \\ 1 - 0.2t &= 0 \\ t &= 5 \quad \text{1M} \end{aligned}$$

$t$	$0 \leq t < 5$	$t = 5$	$5 < t \leq 24$
$\frac{d^2R}{dt^2}$	+	0	-

1M (for testing)

$\therefore \frac{dR}{dt}$  is increasing for  $0 \leq t \leq 5$  and

$\frac{dR}{dt}$  is decreasing for  $5 \leq t \leq 24$ .

對  $0 \leq t \leq 5$ ， $\frac{dR}{dt}$  遞增及

對  $5 \leq t \leq 24$ ， $\frac{dR}{dt}$  遞減。

提示

描述變化時，須使用「increasing (遞增)」及「decreasing (遞減)」這些字眼。

→ 乙部應試特訓 1 : Q1 - Q2

Define  $h(x) = \frac{12x^5}{1+5x^6}$  for all real values of  $x$ .

對所有實數  $x$ ，定義  $h(x) = \frac{12x^5}{1+5x^6}$ 。

(a) Solve the equation  $h'(x) = 0$ .

解方程  $h'(x) = 0$ .

(2 marks)

(b) Does  $h(x)$  attain its extreme value at  $x = 0$ ? Explain your answer. **應試技巧 3.2**

$h(x)$  是否於  $x = 0$  處達至其極值？解釋你的答案。

(1 mark)

(c) Find the greatest value and the least value of  $h(x)$ . **應試技巧 3.3**

求  $h(x)$  的最大值及最小值。

(5 marks)

(d) Find the range of values of  $x$  such that the function of the slope of the tangent to the curve  $y = h(x)$  is increasing for  $x > 0$ .

對  $x > 0$ ，求  $x$  的值使  $y = h(x)$  的切線的斜率為遞增函數。

(4 marks)

←————— 題解 —————→

$$(a) \quad h'(x) = 12 \times \frac{(1+5x^6)(5x^4) - x^5(30x^5)}{(1+5x^6)^2}$$

$$= 12 \times \frac{5x^4(1+5x^6 - 6x^6)}{(1+5x^6)^2}$$

$$= \frac{60x^4(1-x^6)}{(1+5x^6)^2}$$

$$h'(x) = 0$$

$$\frac{60x^4(1-x^6)}{(1+5x^6)^2} = 0$$

1M

$$x^4 = 0 \text{ or } x^6 = 1$$

$$\therefore x = \underline{0} \text{ or } x = \underline{-1} \text{ or } x = \underline{1}$$

1A

(b)	$x$	$-1 < x < 0$	$x = 0$	$0 < x < 1$
	$h'(x)$	+	0	+

$\therefore h(x)$  does not attain its extreme value at  $x = 0$ . 1A ◀ 見 3C 應試備忘。

$h(x)$  在  $x = 0$  處沒有達至其極值。

(c)	$x$	$x < -1$	$x = -1$	$-1 < x < 0$
	$h'(x)$	-	0	+

∴  $h(x)$  attains its minimum value at  $x = -1$ .

The minimum value of  $h(x) = h(-1) = -2$

$h(x)$  於  $x = -1$  達至其極小值。

$h(x)$  的極小值  $= h(-1) = -2$

	$x$	$0 < x < 1$	$x = 1$	$x > 1$
	$h'(x)$	+	0	-

1M (for testing, either one)

∴  $h(x)$  attains its maximum value at  $x = 1$ .

The maximum value of  $h(x) = h(1) = 2$

$h(x)$  於  $x = 1$  達至其極大值。

$h(x)$  的極大值  $= h(1) = 2$

$$\lim_{x \rightarrow +\infty} h(x)$$

1M (either one)

$$= \lim_{x \rightarrow +\infty} \frac{12x^5}{1 + 5x^6}$$

$$= \lim_{x \rightarrow +\infty} \frac{\frac{12}{x}}{\frac{1}{x^6} + 5}$$

$$= \frac{0}{0 + 5}$$

$$= 0$$

$$\lim_{x \rightarrow -\infty} h(x)$$

$$= 0$$

$$\therefore -2 < 0 < 2$$

∴ The greatest value of  $h(x) = h(1) = 2$  1A

and the least value of  $h(x) = h(-1) = -2$ . 1A

$h(x)$  的最大值  $= h(1) = 2$

及  $h(x)$  的最小值  $= h(-1) = -2$ 。

### 提示

由一階導數判別法求得的極大值 (maximum value) 及極小值 (minimum value)，均是局部極值 (local extremum)，它們未必是最大值 (greatest value) 及最小值 (least value)。判別最大值及最小值，須考慮函數的整個定義域。在本例題，定義域為所有實數  $x$ ，我們檢查兩端的極限。

**分析**

- (d) 切線的斜率為  $h'(x)$ 。斜率遞增，表示  $\frac{d}{dx}h'(x) > 0$ ，即  $h''(x) > 0$ 。

$$h'(x) = \frac{60x^4(1-x^6)}{(1+5x^6)^2}$$

$$= 60 \times \frac{x^4 - x^{10}}{(1+5x^6)^2}$$

$$h''(x) = 60 \times \frac{(1+5x^6)^2(4x^3 - 10x^9) - (x^4 - x^{10}) \times 2(1+5x^6)(30x^5)}{(1+5x^6)^4}$$

$$= 60 \times \frac{(1+5x^6)(4x^3 - 10x^9) - (x^4 - x^{10}) \times 60x^5}{(1+5x^6)^3} \quad 1M$$

$$= 60 \times \frac{4x^3 - 50x^9 + 10x^{15}}{(1+5x^6)^3}$$

$$= \frac{120x^3(2 - 25x^6 + 5x^{12})}{(1+5x^6)^3}$$

For  $h''(x) > 0$ ,

對  $h''(x) > 0$ ,

$$\frac{120x^3(2 - 25x^6 + 5x^{12})}{(1+5x^6)^3} > 0$$

1M

$$5x^{12} - 25x^6 + 2 > 0$$

$\because x > 0$   
 $\therefore x^3 > 0$  及  $(1+5x^6)^3 > 0$

$$\therefore x^6 < \frac{25 - \sqrt{585}}{10} \text{ or } x^6 > \frac{25 + \sqrt{585}}{10}$$

1M

$$\text{i.e. } 0 < x < \left(\frac{25 - \sqrt{585}}{10}\right)^{\frac{1}{6}} \text{ or } x > \left(\frac{25 + \sqrt{585}}{10}\right)^{\frac{1}{6}}$$

1A

$\because x > 0$   
 $\therefore$  捨去所有負數的  $x$  值。

→ 乙部應試特訓 1 : Q3 - Q6

## 乙部 應試特訓 1

Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

除特別指明外，數值答案須用真確值或四位小數表示。

1. The average value of flats  $V$  (in ten thousand dollars) in a certain city in the first six months of a certain year can be modelled by  $V = \frac{800}{1 + \alpha t e^{1+\beta t}}$ , where  $t$  ( $0 \leq t \leq 6$ ) is the number of months elapsed since the start of the year, and  $\alpha$  and  $\beta$  are constants.

(a) Express  $\ln\left(\frac{800}{Vt} - \frac{1}{t}\right)$  as a linear function of  $t$ .  
(2 marks)

(b) It is given that the slope and the intercept on the vertical axis of the graph of the linear function obtained in (a) are  $-0.2$  and  $\ln\frac{e}{200}$  respectively.

- (i) Find  $\alpha$  and  $\beta$ .  
(ii) Will the average value of flats be less than 780 ten thousand dollars during the first 6 months in the year? Explain your answer.  
(iii) Describe how the rate of change of the average value of flats in the city varies during the first six months in the year. Explain your answer.

(10 marks)

某年的上半年，某城市樓宇每月的平均價格  $V$  (以萬元為單位) 可用  $V = \frac{800}{1 + \alpha t e^{1+\beta t}}$  來模擬，其中  $t$  ( $0 \leq t \leq 6$ ) 為自該年年初開始起計所經過的月數，而  $\alpha$  及  $\beta$  均為常數。

(a) 將  $\ln\left(\frac{800}{Vt} - \frac{1}{t}\right)$  表為  $t$  的線性函數。  
(2 分)

(b) 已知在 (a) 所得的線性函數的圖像的斜率及垂直軸上的截距分別為  $-0.2$  及  $\ln\frac{e}{200}$ 。

- (i) 求  $\alpha$  及  $\beta$ 。  
(ii) 該城市樓宇在該上半年每月的平均價格是否能小於 780 萬元？試解釋你的答案。  
(iii) 描述該城市樓宇在該上半年每月的平均價格的變率如何變化。試解釋你的答案。

(10 分)

參考：2016Q12

2. The area  $A$  (in  $\text{m}^2$ ) covered by a certain pollutant can be modelled by

$$A(t) = \frac{300}{2 + ae^{-bt}},$$

where  $a$  and  $b$  are positive constants and  $t$  ( $0 \leq t \leq 120$ ) is the time passed (in minutes) since the pollutant exposed in the air. The table below shows the data collected.

$t$	5	10	15	20
$A(t)$	6.69	8.48	10.72	13.49

- (a) Express  $\ln\left(\frac{300}{A(t)} - 2\right)$  as a linear function of  $t$ .  
(2 marks)
- (b) Using the graph paper on p. 75, estimate the values of  $a$  and  $b$ . (Give the answers correct to 2 significant figures if necessary.)  
(3 marks)
- (c) (i) Find  $A'(t)$  and  $A''(t)$ .  
(ii) Will the area covered by the pollutant exceed  $156 \text{ m}^2$ ? Explain your answer.  
(6 marks)
- (d) Hence, describe how  $A'(t)$  varies with time  $t$ .  
(2 marks)

某污染物所覆蓋的面積  $A$  (以  $\text{m}^2$  為單位) 可用下式模擬：

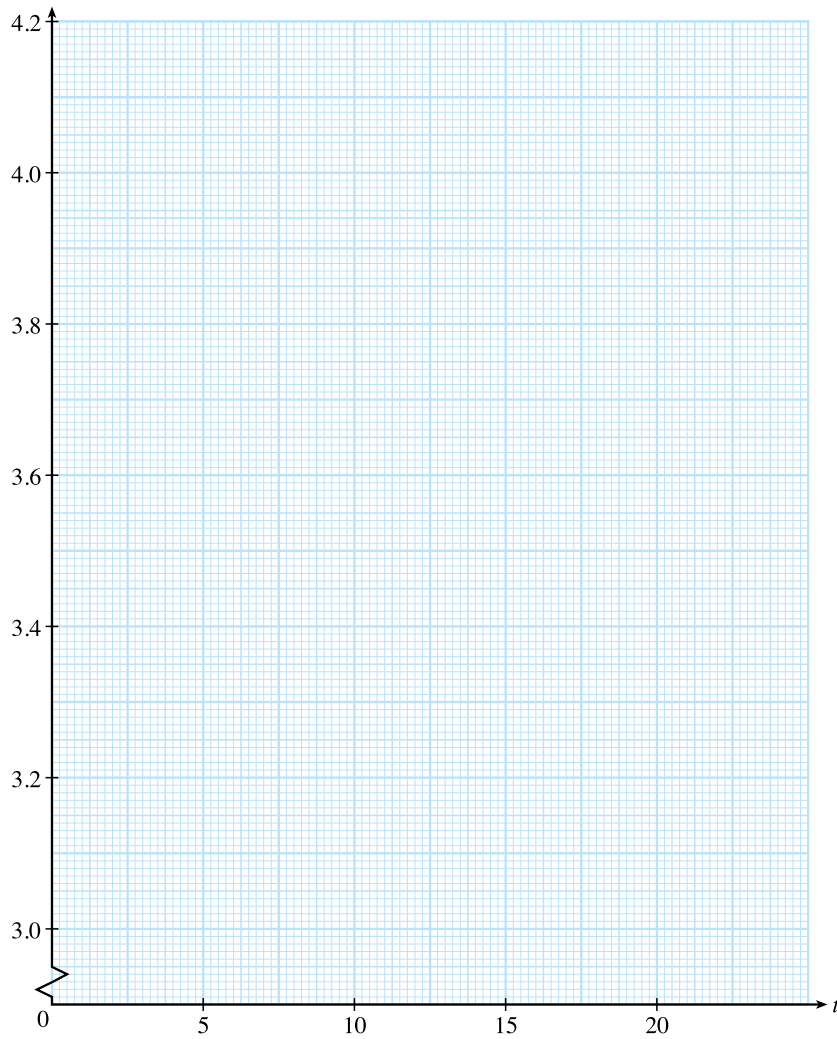
$$A(t) = \frac{300}{2 + ae^{-bt}},$$

其中， $a$  及  $b$  均為正常數且  $t$  ( $0 \leq t \leq 120$ ) 為污染物開始曝露於空氣中起計所經過的時間 (以分鐘為單位)。下表顯示了收集的數據。

$t$	5	10	15	20
$A(t)$	6.69	8.48	10.72	13.49

- (a) 將  $\ln\left(\frac{300}{A(t)} - 2\right)$  表為  $t$  的線性函數。  
(2 分)
- (b) 利用第 75 頁的方格紙，估算  $a$  和  $b$  的值。(如有需要，答案須準確至二位有效數字。)  
(3 分)
- (c) (i) 求  $A'(t)$  及  $A''(t)$ 。  
(ii) 污染物所覆蓋的面積是否能大於  $156 \text{ m}^2$ ? 試解釋你的答案。  
(6 分)
- (d) 由此，描述  $A'(t)$  隨時間的變化。  
(2 分)

$$\ln\left(\frac{300}{A(t)} - 2\right)$$



1

2

3

**B(I)**

4

5

B

6

7

B

8

B

A

3. A promotion company creates a video channel for a singer. Define  $u = e^{12-6t}$ , where  $t$  ( $t \geq 0$ ) is the number of days elapsed since the start of the creation of the channel. The company models the total number of subscribers  $N$  of the channel by  $N = ke^{-u}$ , where  $k$  is a positive constant.

(a) Find  $\frac{du}{dt}$ . Hence, express  $\frac{dN}{dt}$  in terms of  $u$ . (3 marks)

(b) Find a polynomial  $g(u)$  in  $u$  such that  $\frac{d^2N}{dt^2} = N \cdot g(u)$ . (2 marks)

(c) It is given that  $\frac{dN}{dt}$  attains its extreme value when  $t = t_0$ .

(i) Find  $t_0$ .

(ii) Determine whether the extreme value of  $\frac{dN}{dt}$  is a maximum value or a minimum value. Explain your answer. (4 marks)

(d) Estimate the total number of subscribers of the channel, in terms of  $k$ , after a very long time. (2 marks)

某宣傳公司為某歌手設立了一個影片頻道。定義  $u = e^{12-6t}$ ，其中  $t$  ( $t \geq 0$ ) 為自該頻道開始設立起計所經過的日數。該公司用  $N = ke^{-u}$ ，其中  $k$  為一正常數，模擬該頻道的訂閱人數。

(a) 求  $\frac{du}{dt}$ 。由此，以  $u$  表  $\frac{dN}{dt}$ 。 (3 分)

(b) 求  $u$  的多項式  $g(u)$  使得  $\frac{d^2N}{dt^2} = N \cdot g(u)$ 。 (2 分)

(c) 已知當  $t = t_0$  時， $\frac{dN}{dt}$  達至其極值。

(i) 求  $t_0$ 。

(ii) 判別  $\frac{dN}{dt}$  的極值是一極大值或一極小值。解釋你的答案。 (4 分)

(d) 估計經過一段很長時間後該頻道的訂閱人數，答案以  $k$  表示。 (2 分)

參考：2022Q12



4. In a medical treatment, the concentration  $A(t)$  (in mg/L) of a certain chemical in the blood is measured  $t (> 0)$  hours after a drug is injected into a patient. The concentration of the chemical can be modelled by

$$A(t) = 4a + 6e^{-kt} - (2a + 6)e^{-2kt},$$

where  $a$  and  $k$  are positive constants. It is known that  $A(0) = 0.5$  and  $A(b) = A(b + 2.5) = 1.5$ , where  $b$  is a positive constant.

- (a) Find  $a$ .  
(1 mark)
- (b) Find the greatest concentration of the chemical in the blood.  
(4 marks)
- (c) Find the exact value of  $k$ .  
(4 marks)
- (d) Someone claims that the concentration of the chemical in the blood will drop below 0.75 mg/L after attaining the greatest value. Is the claim correct? Explain your answer.  
(3 marks)

在一次藥物治療中，病人在注射後的  $t (> 0)$  小時，某化學物質在血液的濃度  $A(t)$  (以 mg/L 為單位) 可用下式模擬：

$$A(t) = 4a + 6e^{-kt} - (2a + 6)e^{-2kt},$$

其中  $a$  及  $k$  均為正常數。已知  $A(0) = 0.5$  及  $A(b) = A(b + 2.5) = 1.5$ ，其中  $b$  為一正常數。

- (a) 求  $a$ 。  
(1 分)
- (b) 求該化學物質在血液的最大濃度。  
(4 分)
- (c) 求  $k$  的真確值。  
(4 分)
- (d) 某人宣稱該化學物質在血液達至最大濃度後便會下降至低於 0.75 mg/L。該宣稱是否正確？試解釋你的答案。  
(3 分)

5. A shop produces cups with special design. The cup is a right prism with a square base with side length  $x$  cm and height  $h$  cm. It is given that the capacity of the cup is  $k$  cm<sup>3</sup>, where  $k$  is a positive constant.

The cost  $\$C$  of producing a cup is a sum of two parts: for the square base, the cost is  $\$0.05/\text{cm}^2$ ; for the other four lateral faces, the cost is  $\$0.01/\text{cm}^2$ .

(a) (i) Express  $h$  in terms of  $x$  and  $k$ .

(ii) Hence, express  $C$  in terms of  $x$  and  $k$ .

(2 marks)

(b) (i) Find  $x$  in terms of  $k$  if  $C$  attain its least value.

(ii) Find  $k$  if the least cost of producing a cup is  $\$0.6$ .

(6 marks)

- (c) Suppose that the cost of the square base and the cost of the other four lateral faces are both increased by 100%. Michael, one of the shopkeepers, claims that the result in (b)(i) will have the same percentage increase. Do you agree? Explain your answer.

(2 marks)

某商店生產一款特別設計的杯子。該杯子是一個以正方形為底的直立角柱體，其底為邊長  $x$  cm 的正方形，高為  $h$  cm。已知該杯子的容量為  $k$  cm<sup>3</sup>，其中  $k$  為一正常數。生產一個杯子的成本  $\$C$  可分為兩部分：正方形底的成本為  $\$0.05/\text{cm}^2$ ，而另外四個側面的成本為  $\$0.01/\text{cm}^2$ 。

(a) (i) 以  $x$  及  $k$  表  $h$ 。

(ii) 由此，以  $x$  及  $k$  表  $C$ 。

(2 分)

(b) (i) 求  $C$  為最小值時的  $x$ ，答案以  $k$  表示。

(ii) 若生產一個杯子的最低成本為  $\$0.6$ ，求  $k$ 。

(6 分)

- (c) 假設正方形底的成本及四個側面的成本均增加了 100%。一位售貨員浩銘宣稱 (b)(i) 的結果也會有相同的百分增加。你是否同意？試解釋你的答案。

(2 分)

6. Define  $f(x) = -(x^4 + x^3 + 4x^2 + 11x + 9)e^{-x}$  for  $0 \leq x \leq 10$ .

(a) Solve the equation  $f'(x) = 0$ .

(2 marks)

(b) Does  $f(x)$  attain its extreme value at  $x = 1$ ? Explain your answer.

(1 mark)

(c) Find the greatest value and the least value of  $f(x)$ .

(4 marks)

(d) Find the equation of the tangent with the greatest slope for the curve  $y = f(x)$ , where  $2 \leq x \leq 6$ .

(5 marks)

對  $0 \leq x \leq 10$ ，定義  $f(x) = -(x^4 + x^3 + 4x^2 + 11x + 9)e^{-x}$ 。

(a) 解方程  $f'(x) = 0$ 。

(2 分)

(b)  $f(x)$  是否於  $x = 1$  處達至其極值？解釋你的答案。

(1 分)

(c) 求  $f(x)$  的最大值及最小值。

(4 分)

(d) 求曲線  $y = f(x)$  的最大斜率的切線的方程，其中  $2 \leq x \leq 6$ 。

(5 分)

參考：2023Q11

1

2

3

B(I)

4

5

B

6

7

B

8

B

A